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The origins of calculus

Goblets modeling task

Try this worksheet after you have completed Exercise 7M.

As a renowned mathemagician and graduate of wizardry, you have been commissioned to design four goblets to be awarded at a tournament.

The goblet specifications are:

- They must be a solid of revolution.
- They must all hold the same volume of liquid.
- The height of the bowl must be 1 cm greater than the height of the liquid.
- The ratio of the height of the center of mass to the radius of the base must be no more than 3:1
- The radius of the stem must be at least 0.25 cm at its thinnest point.

By researching optimal goblet shapes add your own specifications for the following:

- The 4th prize goblet is to be a water goblet.
- The 3rd prize goblet is to contain white witches brew. This brew is served chilled, so the goblet bowl needs to be narrower, to keep the liquid cool.
- The 2nd prize goblet is to contain red witches brew. This brew is served at room temperature, so the goblet bowl needs to be wider for the full taste to develop.
- The 1st prize goblet will contain a steamy, frothy, spiced red witches brew.

Finding the center of mass of a solid of revolution

The center of mass, or centroid, of an object is the point at which the object will be horizontally balanced if suspended from that point. In the case of glasses filled with fluid, you don't want the glass to tip over when full so this is an important consideration.

The x -coordinate of the center of mass of a **solid of revolution** of region M about

the x -axis is $x = \frac{\int_a^b \pi x f(x)^2 dx}{\int_a^b \pi f(x)^2 dx}$, and the y -coordinate is 0.

You could create one model whose parameters can be changed in meeting the specific uses of the different goblets. You might want to consider the major axis of its profile as lying on the x -axis, and work with piecewise functions to define the different shapes along the goblet from its stem to the cup itself. Using the x -axis enables you to consider different shapes that can then be revolved about the axis to create a solid of revolution.