

# Using a graphic display calculator

## CHAPTER OBJECTIVES:

This chapter shows you how to use your graphic display calculator (GDC) to solve the different types of problems that you will meet in your course. You should not work through the whole of the chapter – it is simply here for reference purposes. When you are working on problems in the mathematical chapters, you can refer to this chapter for extra help with your GDC if you need it.

Instructions for the TI-84 Plus calculator

Use this list to help you to find the topic you need

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## Before you start

### You should be familiar with:

- Important keys on the keyboard: **ON** **2nd** **DEL** **CLEAR** **Y=** **X, T,  $\theta$ , n** **ENTER** **GRAPH**
- The home screen
- Changing window settings in the graph screen
- Using zoom tools in the graph screen
- Using trace in the graph screen

For a reminder of how to perform the basic operations have a look at your GDC manual.

## 1 Functions

### 1.1 Graphing linear functions

#### Example 1

Draw the graph of the function  $y = 2x + 1$ .

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

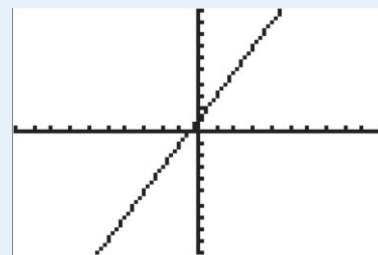
Type  $2x + 1$  and press **ENTER**.

Press **ZOOM** | 6:ZStandard to use the default axes which are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .

```

Plot1 Plot2 Plot3
Y1=2X+1
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
  
```

The graph of  $y = 2x + 1$  is now displayed on the screen.



### Finding information about the graph

The GDC can give you a lot of information about the graph of a function, such as the coordinates of points of interest and the gradient (slope).

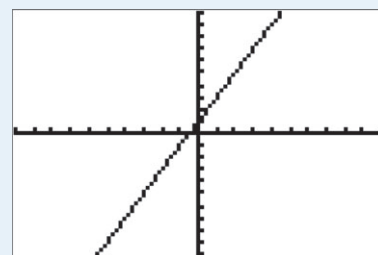
### 1.2 Finding a zero

The  $x$ -intercept is known as a *zero* of the function.

#### Example 2

Find the zero of  $y = 2x + 1$ .

Draw the graph of  $y = 2x + 1$  as in Example 1.



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Press **2nd** **CALC** | 2:Zero.

Press **ENTER**.

```

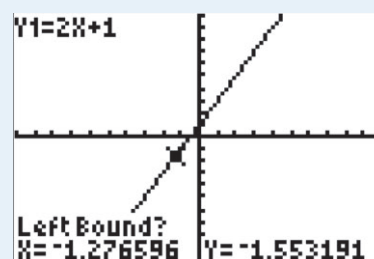
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

To find the zero you need to give the left and right bounds of a region that includes the zero.

The calculator shows a point and asks you to set the left bound.

Move the point using the **◀** and **▶** keys to choose a position to the left of the zero.

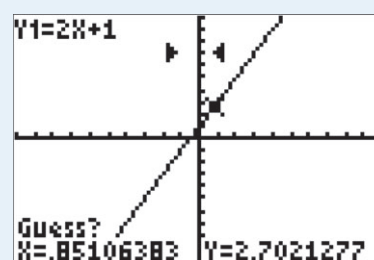
Press **ENTER**.



The calculator shows another point and asks you to set the right bound.

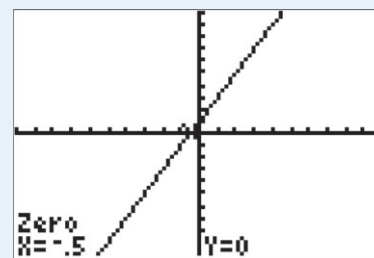
Move the point using the **◀** and **▶** keys so that the region between the left and right bounds contains the zero.

When the region contains the zero press **ENTER**.



Press **ENTER** again to supply a guess for the value of the zero.

The calculator displays the zero of the function  $y = 2x + 1$  at the point  $(-0.5, 0)$ .



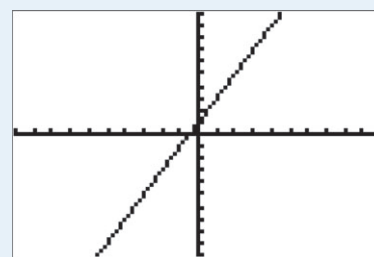
### 1.3 Finding the gradient (slope) of a line

The correct mathematical notation for gradient (slope) is  $\frac{dy}{dx}$ . You will find out more about this in the chapter on differential calculus. Here we just need to know this is the notation that will give us the gradient (slope) of the line.

#### Example 3

Find the gradient of  $y = 2x + 1$ .

First draw the graph of  $y = 2x + 1$  as in Example 1.



▶ Continued on next page

Press **2nd** **CALC** | 6:  $dy/dx$ .

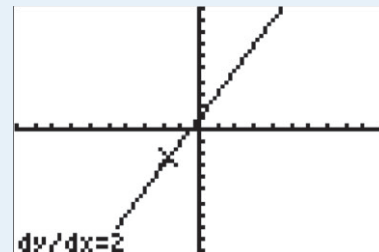
Press **ENTER**.

```

CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
  
```

Select any point on the line using the **◀** and **▶** keys and press **ENTER**.

The gradient (slope) is 2.



## 1.4 Solving simultaneous equations graphically

To solve simultaneous equations graphically you draw the straight lines and then find their point of intersection. The coordinates of the point of intersection give you the solutions  $x$  and  $y$ .

**Note:** The calculator will only draw the graphs of functions that are expressed explicitly. By that we mean functions that begin with ' $y =$ ' and have a function that involves only  $x$  to the right of the equals sign. If the equations are written in a different form, you will need to rearrange them before using your calculator to solve them.

Solving simultaneous equations using a non-graphical method is covered in section 1.5.

### Example 4

Solve the simultaneous equations  $2x + y = 10$  and  $x - y = 2$  graphically with your GDC.

First rearrange both equations in the form  $y =$

$$\begin{array}{ll}
 2x + y = 10 & x - y = 2 \\
 y = 10 - 2x & -y = 2 - x \\
 & y = x - 2
 \end{array}$$

To draw graphs  $y = 10 - 2x$  and  $y = x - 2$ .

Press **Y=** to display the Y= editor. The default graph type is Function, so the form  $Y=$  is displayed.

Type  $10 - 2x$  and press **ENTER** and  $x - 2$  and press **ENTER**.

Press **ZOOM** | 6:Z Standard to use the default axes which are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .

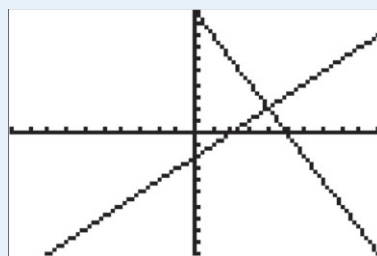
```

Plot1 Plot2 Plot3
Y1=10-2X
Y2=X-2
Y3=
Y4=
Y5=
Y6=
Y7=
  
```

The calculator displays both straight line graphs

$Y1 = 10 - 2x$  and

$Y2 = x - 2$



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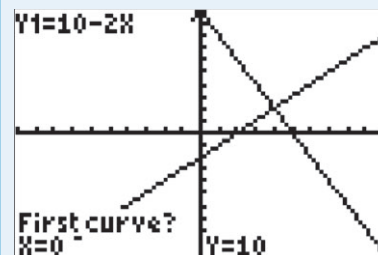


Press **2nd** **CALC** | 5:intersect.

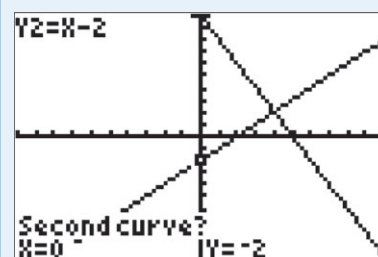
Press **ENTER**.

**CALCULATE**  
 1:value  
 2:zero  
 3:minimum  
 4:maximum  
 5:intersect  
 6:dy/dx  
 7:∫f(x)dx

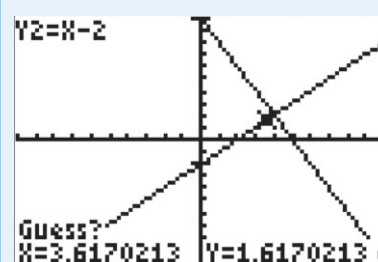
Press **ENTER** to select the first curve.



Press **ENTER** to select the second curve.

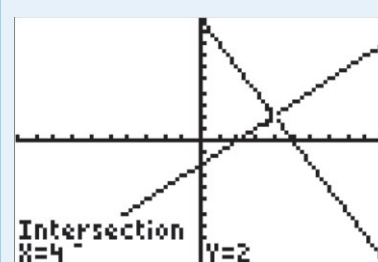


Select a point close to the intersection using the **◀** and **▶** keys and press **ENTER**.



The calculator displays the intersection of the two straight lines at the point (4, 2).

The solutions are  $x = 4$ ,  $y = 2$ .



# Simultaneous and quadratic equations

## 1.5 Solving simultaneous linear equations in two unknowns

When solving simultaneous equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The calculator will do all the working for you.

You will need to have the App PlySmlt2 installed on your GDC. This App is permitted by IBO in your examination.

### Example 5

Solve the equations:

$$2x + y = 10$$

$$x - y = 2$$

Press **APPS**. You will see the dialog box as shown on the right. Choose the App PlySmlt2 and press **ENTER**.

```
APPLICATIONS
1: Finance...
2: CtlgHelp
3: PlySmlt2
```

From the main menu, choose 2: SIMULT EQN SOLVER and press **ENTER**.

```
MAIN MENU
1: POLY ROOT FINDER
2: SIMULT EQN SOLVER
3: ABOUT
4: POLY HELP
5: SIMULT HELP
6: QUIT POLYSMLT
```

The defaults are to solve two equations in two unknowns.

**Note:** This is how you will use the linear equation solver in your examinations. In your project, you might want to solve a more complicated system with more equations and more variables.

```
SIMULT EQN SOLVER MODE
EQUATIONS 3 4 5 6 7 8 9 10
UNKNOWN 2 3 4 5 6 7 8 9 10
DEC FRAC
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
(MAIN) (HELP) (NEXT)
```

Press **F5** and you will see the template on the right.

Type the coefficients from two equations into the template, pressing **ENTER** after each number.

The equations must be in the correct order.

```
SYSTEM MATRIX (2x3)
[0 0 | 0 ]
[0 0 | 0 ]

(1,1)=0
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)
```

Press **F5** and the calculator will solve the equations, giving the solutions in the as  $x_1$  and  $x_2$ .

```
SYSTEM MATRIX (2x3)
[2 1 | 10 ]
[1 -1 | 2 ]

(2,3)=2
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)
```

▶ Continued on next page

The solutions are  $x = 4$ ,  $y = 2$ .

```

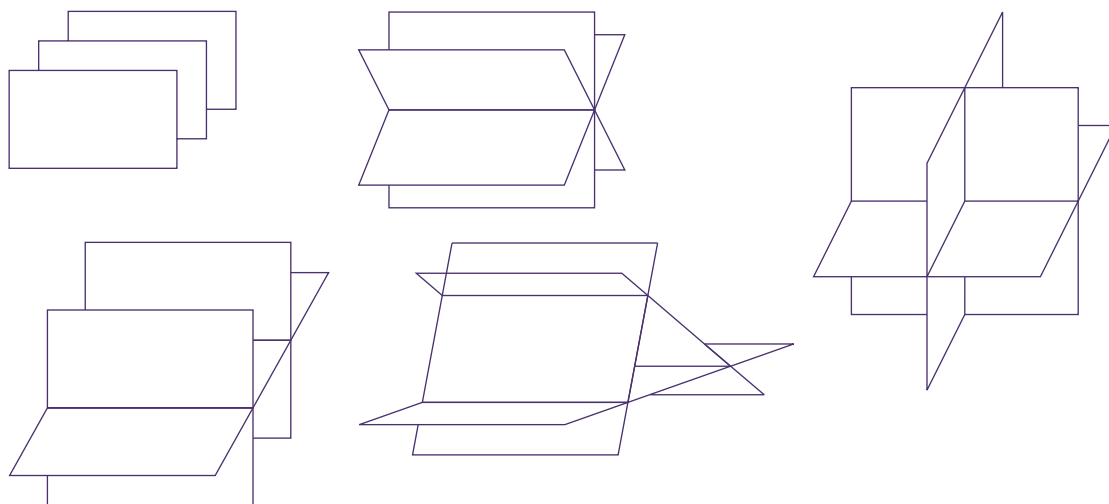
SOLUTION
x1 = 4
x2 = 2

[MAIN][MODE][SYSM][STD][F4][D]

```

## 1.6 Solving simultaneous equations in three unknowns

When solving simultaneous equations in three unknowns there might be a unique solution, infinitely many solutions or no solutions at all. Geometrically, if the equations represent planes in three-dimensions, then their solutions would be intersection at a point, intersection on a line (or plane) or non-intersecting planes.



### Example 6

Solve the equations

$$2x - 3y + 4z = 1$$

$$x - y - z = -1$$

$$-x + 2y - z = 2$$

Press **APPS**. You will see the dialog box as shown on the right.  
Choose the App PlySmlt2 and press **ENTER**.

```

APPLICATIONS
1: Finance...
2: CtlgHelp
3: PlySmlt2

```

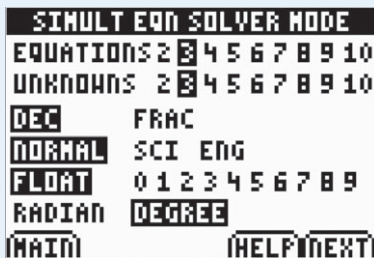
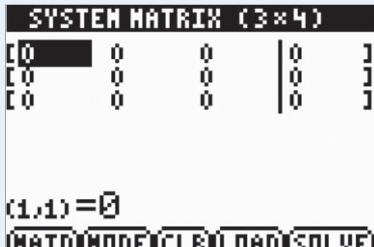
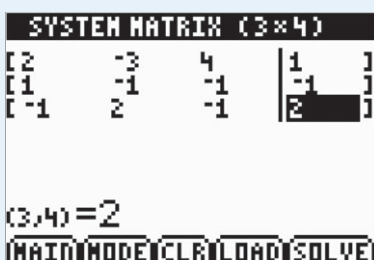
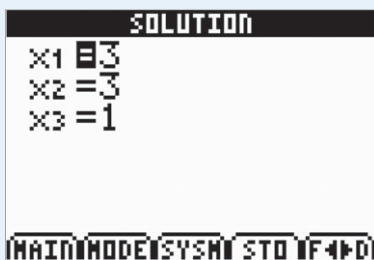
From the main menu, choose 2: SIMULT EQN SOLVER and press **ENTER**.

```

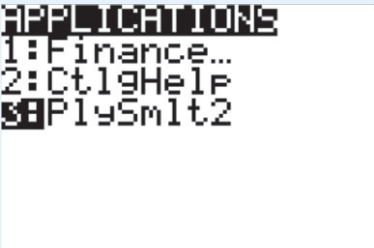

MAIN MENU
1: POLY ROOT FINDER
2: SIMULT EQN SOLVER
3: ABOUT
4: POLY HELP
5: SIMULT HELP
6: QUIT POLYSMLT

```

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Change the defaults to three equations in three unknowns.	
Press <b>F5</b> and you will see the template on the right. Type the coefficients from two equations into the template, pressing <b>ENTER</b> after each number.	
Press <b>F5</b> and the calculator will solve the equations, giving the solutions as $x_1$ , $x_2$ and $x_3$ .	
The solutions are $x = 3$ , $y = 3$ and $z = 1$ . In this example, the solutions represent a point.	

## Example 7

Solve the equations $2x + 4y + 2z = 8$ $x + 2y + z = 4$ $3x - y + z = -9$	
Press <b>APPS</b> . You will see the dialog box as shown on the right. Choose the App PlySmlt2 and press <b>ENTER</b> .	
From the main menu, choose 2: SIMULT EQN SOLVER and press <b>ENTER</b> .	

► Continued on next page

Change the defaults to three equations in three unknowns.

```
SIMULT EQN SOLVER MODE
EQUATIONS 2 3 4 5 6 7 8 9 10
UNKNOWN 2 3 4 5 6 7 8 9 10
DEC      FRAC
NORMAL   SCI  ENG
FLOAT    0 1 2 3 4 5 6 7 8 9
RADIAN   DEGREE
(MAIN)   (HELP) (NEXT)
```

Press **F5** and you will see the template on the right.

Type the coefficients from two equations into the template, pressing **ENTER** after each number.

The equations must be in the correct order.

```
SYSTEM MATRIX (3x4)
[0 0 0 | 0 ]
[0 0 0 | 0 ]
[0 0 0 | 0 ]

(1,1)=0
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)
```

Press **F5** and the calculator will solve the equations, giving the solutions as  $x_1$ ,  $x_2$  and  $x_3$ .

```
SYSTEM MATRIX (3x4)
[2 4 2 | 8 ]
[1 2 1 | 4 ]
[3 -1 1 | -9 ]

(3,4) = -9
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)
```

The solutions are  $x = -2 - \frac{3x_3}{7}$ ,  $y = 3 - \frac{2x_3}{7}$  and  $z = x_3$ .

In this example, the solutions represent a straight line.

Since  $z = x_3$  (an arbitrary constant) the equations of the line can be written

$$\frac{7(x+2)}{-3} = \frac{7(y-3)}{-2} = z$$

$$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z}{-7}$$

```
SOLUTION SET
X1 = -2-3/7X3
X2 = 3-2/7X3
X3 = X3

(MAIN) (MODE) (SYSM) (STD) (VREF)
```

## Example 8

Solve the equations

$$x + 2y - 3z = 13$$

$$2x - y + x = 4$$

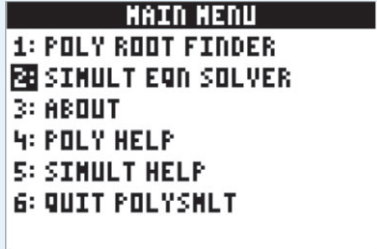
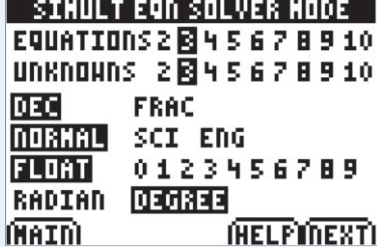
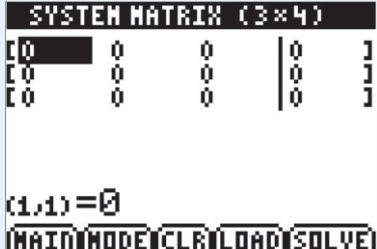


$$x + 2y - 3z = 7$$

Press **APPS**. You will see the dialog box as shown on the right.

Choose the App PlySmlt2 and press **ENTER**.

```
APPLICATIONS
1: Finance...
2: CtlgHelp
3: PlySmlt2
```

► Continued on next page

<p>From the main menu, choose 2: SIMULT EQN SOLVER and press <b>ENTER</b>.</p>	
<p>Change the defaults to three equations in three unknowns.</p>	
<p>Press <b>F5</b> and you will see the template on the right. Type the coefficients from the three equations into the template, pressing <b>ENTER</b> after each number.</p> <p>The equations must be in the correct order.</p>	
<p>Press <b>F5</b> and the calculator will solve the equations, giving the solutions as <math>x_1</math>, <math>x_2</math> and <math>x_3</math>.</p>	
<p>There are no solutions. In this example the equations are inconsistent.</p>	



## Quadratic functions

### 1.7 Drawing a quadratic graph

#### Example 9

Draw the graph of  $y = x^2 - 2x + 3$  and display it using suitable axes.

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type  $x^2 - 2x + 3$  and press **ENTER**.

Press **ZOOM** | 6:Z Standard to use the default axes which are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .

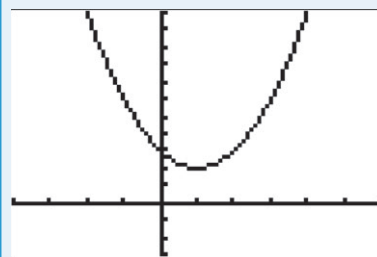
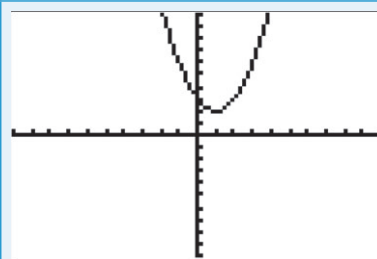
The calculator displays the curve with the default axes.

Adjust the window to make the quadratic curve fit the screen better.

For help with changing axes, see your GDC manual.

```

P1ot1 P1ot2 P1ot3
\Y1=X^2-2X+3
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
  
```



### 1.8 Solving quadratic equations

When solving quadratic equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

#### Example 10

Solve  $3x^2 - 4x - 2 = 0$

Press **APPS**. You will see the dialog box as shown on the right. Choose the App PlySmlt2 and press **ENTER**.

```

APPLICATIONS
1: Finance...
2: CtlgHelp
3: PlySmlt2
  
```

From the main menu, choose 1: POLY ROOT FINDER and press **ENTER**.

```

MAIN MENU
1: POLY ROOT FINDER
2: SIMULT EQN SOLVER
3: ABOUT
4: POLY HELP
5: SIMULT HELP
6: QUIT POLYSMLT
  
```

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The defaults are to solve an equation of order 2 (a quadratic equation) with real roots. You do not need to change anything.

```

POLY ROOT FINDER MODE
ORDER  1 2 3 4 5 6 7 8 9 10
REAL   a+bi  re^(θi)
DEC     FRAC
NORMAL SCI ENG
FLOAT   0 1 2 3 4 5 6 7 8 9
RADIAN  DEGREE
(MAIN)      (HELP)(NEXT)
  
```

Another dialog box opens for you to enter the equation.

The general form of the quadratic equation is  $a_2x^2 + a_1x + a_0 = 0$ , so we enter the coefficients in  $a_2$ ,  $a_1$  and  $a_0$ .

```

a2x^2+a1x+a0=0
a2 =
a1 =
a0 =
(MAIN)(MODE)(CLR)(LOAD)(SOLVE)
  
```

Here  $a_2 = 3$ ,  $a_1 = -4$  and  $a_0 = -2$ . Be sure to use the  $(-)$  key to enter the negative values.

Press **ENTER** after each value.

Press **F5** and the calculator will find the roots of the equation.

```

a2x^2+a1x+a0=0
a2 = 3
a1 = -4
a0 = -2
(MAIN)(MODE)(CLR)(LOAD)(SOLVE)
  
```

The solutions are  $x = -0.387$  or  $x = 1.72$  (3 sf).

```

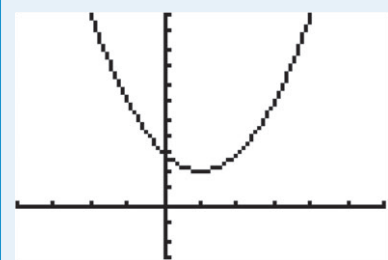
a2x^2+a1x+a0=0
x1 = 1.72075922
x2 = -.3874258867
(MAIN)(MODE)(COEF)(STD)(F4)(D)
  
```

## 1.9 Finding a local minimum or maximum point

### Example 11

Find the minimum point on the graph of  $y = x^2 - 2x + 3$ .

Draw the graph of  $y = x^2 - 2x + 3$  (See Example 9).



#### Method 1 - using a table

You can look at the graph and a table of the values on the graph by using a split screen.

Press **MODE** and select G-T.

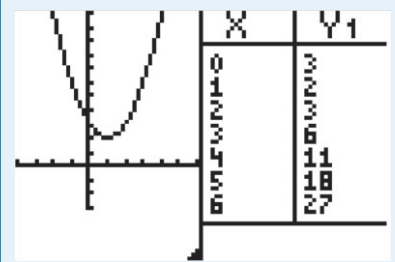
Press **GRAPH**.

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^(θi)
FULL HORIZ G-T
4NEXT+
  
```

▶ Continued on next page

The minimum value shown in the table is 2 when  $x = 1$ .



Look more closely at the values of the function around  $x = 1$ .

Change the settings in the table: Press **2nd** **TBLSET**.

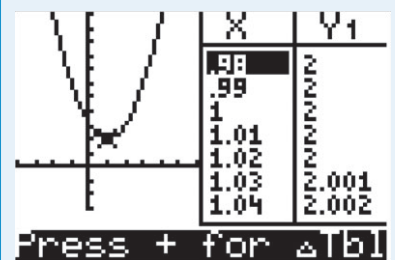
Set TblStart to 0.98

$\Delta$ Tbl to 0.01

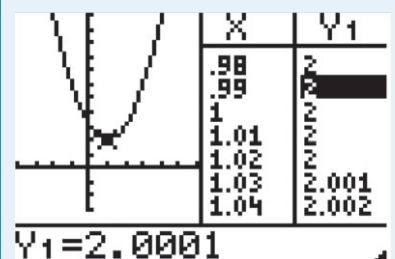
Press **2nd** **TABLE** to return to the graph and table screen.

TABLE SETUP  
TblStart=.98  
 $\Delta$ Tbl=.01  
Indent: Auto Ask  
Depend: Auto Ask

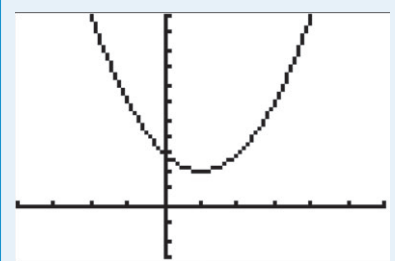
Press **▸** to move to the column containing  $y$ -values. This shows greater precision in the box below the table.



The table shows that the function has larger values at points around (1, 2). We can conclude that this is a local minimum on the curve.



### Method 2 - Using the minimum function



Press **2nd** **CALC** | 3:minimum.

Press **ENTER**.

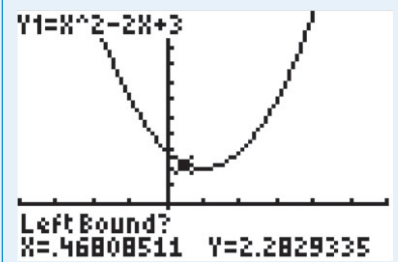
CALCULATE  
1:value  
2:zero  
3:minimum  
4:maximum  
5:intersect  
6:dy/dx  
7: $\int f(x)dx$

► Continued on next page

To find the minimum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the  $\leftarrow$  and  $\rightarrow$  keys to choose a position to the left of the minimum.

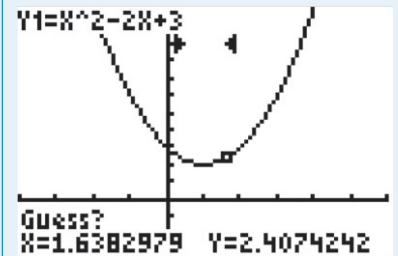
Press **ENTER**.



The calculator shows another point and asks you to set the right bound.

Move the point using the  $\leftarrow$  and  $\rightarrow$  keys so that the region between the left and right bounds contains the minimum.

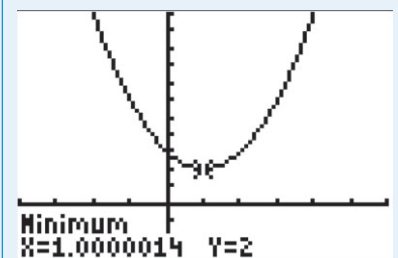
When the region contains the minimum press **ENTER**.



Press **ENTER** again to supply a guess for the value of the minimum.

The calculator displays the minimum point on the curve at (1, 2).

In this example the value of  $x$  is not exactly 1. This is due to the way the calculator finds the point. You should ignore small errors like this when you write down the coordinates of the point.



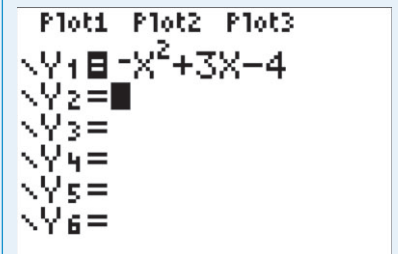
## Example 12

Find the maximum point on the graph of  $y = -x^2 + 3x - 4$ .

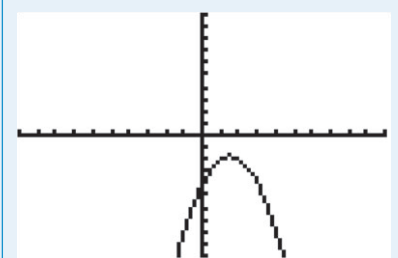
Press **Y=** to display the Y= editor. The default graph type is Function, so the form  $Y=$  is displayed.

Type  $-x^2 + 3x - 4$  and press **ENTER**.

Press **ZOOM** | 6:Z Standard to use the default axes which are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .

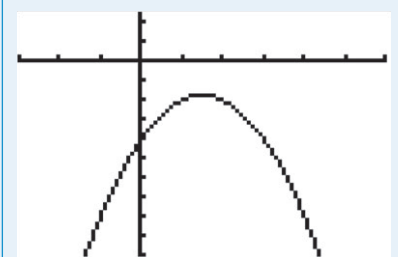


The calculator displays the curve with the default axes.



Adjust the window to make the quadratic curve fit the screen better.

For help with changing axes, see your GDC manual.



► Continued on next page

**Method 1 - using a table**

You can look at the graph and a table of the values on the graph by using a split screen.

Press **MODE** and select G-T.

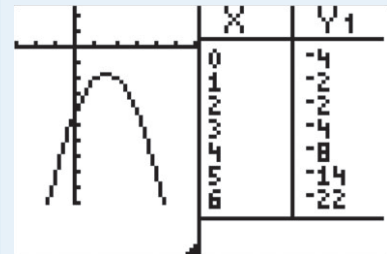
Press **GRAPH**.

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
↓NEXT↓

```

The maximum value shown in the table is  $-2$  when  $x = 1$  and  $x = 2$ .



Look more closely at the values of the function between  $x = 1$  and  $x = 2$ .

Change the settings in the table: Press **2nd** **TBLSET**.

Set TblStart to 1.4

$\Delta$ Tbl to 0.01

Press **2nd** **TABLE** to return to the graph and table screen.

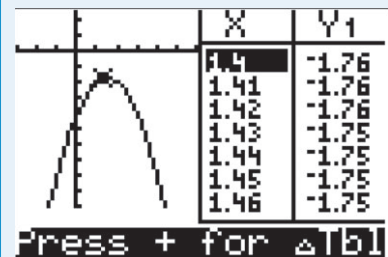
```

TABLE SETUP
TblStart=1.4
ΔTbl=.01
Indent: Auto Ask
Depend: Auto Ask

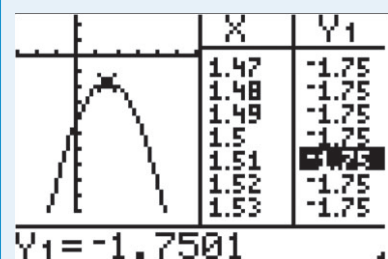
```

Press **▶** to move to the column containing  $y$ -values. This shows greater precision in the box below the table.

Press **▼** to scroll down until you find the maximum value of  $y$ .

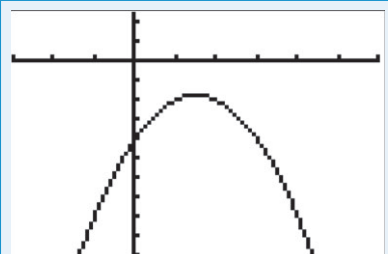


The table shows that the function has smaller values at points around  $(1.5, -1.75)$ . We can conclude that this is a local maximum on the curve.

**Method 2 - Using the maximum function**

Press **2nd** **CALC** | 4:maximum.

Press **ENTER**.



```

CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx

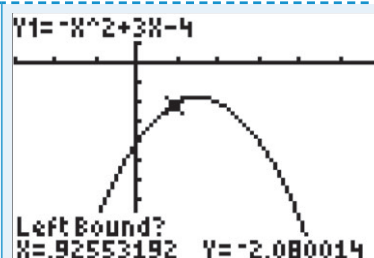
```

▶ Continued on next page

To find the maximum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the  $\leftarrow$  and  $\rightarrow$  keys to choose a position to the left of the maximum.

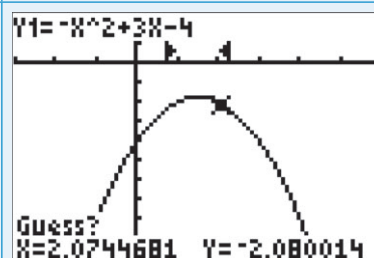
Press **ENTER**.



The calculator shows another point and asks you to set the right bound.

Move the point using the  $\leftarrow$  and  $\rightarrow$  keys so that the region between the left and right bounds contains the minimum.

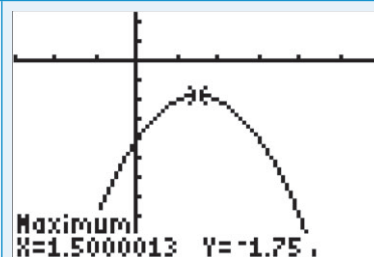
When the region contains the minimum press **ENTER**.



Press **ENTER** again to supply a guess for the value of the minimum.

The calculator displays the maximum point on the curve at (1.5, -1.75).

In this example the value of  $x$  is not exactly 1.5. This is due to the way the calculator finds the point. You should ignore small errors like this when you write down its coordinates.



## Complex numbers

### 1.10 Operations with complex numbers

#### Example 13

Evaluate the following expressions

i  $2(7 + i) + \frac{1}{2}(4 - 2i)$

ii  $(2 + 3i) \cdot (3 - 4i)$

iii  $\sqrt{3 + 4i}$

iv  $\frac{1-i}{3+i}$

v  $(1 - i)^3$

Complex calculations are entered in the same way as you would enter a real expression.

To enter the imaginary symbol  $i$  press number **2nd**  $i$

Enter the expressions and then press **EXE**

The results are as shown.

Note that the fraction template **ALPHA** **F2** does not work with complex numbers.

$$2(7+i) + \frac{1}{2}(4-2i) = 16+i$$

$$(2+3i)(3-4i) = 18+i$$

$$\sqrt{3+4i} = 2+i$$

$$(1-i)/(3+i) = .2-.4i$$

Ans  $\rightarrow$  Frac  $\frac{1}{5} - \frac{2}{5}i$

$$(1-i)^2 = -2i$$



## 1.11 Conjugate, modulus and argument

### Example 14

Let  $z = 1 + \sqrt{3}i$

Find i  $z^*$

ii  $|z|$

iii  $\arg(z)$

i Press **MODE** | CPX | 1:conj(

Enter the complex number.

To enter the imaginary symbol  $i$  press number **SHIFT**  $i$

Press **ENTER**

```
conj(1+√3i)
1-1.732050808i
```

ii Press **MODE** | CPX | 5:abs(

Enter the complex number.

To enter the imaginary symbol  $i$  press number **SHIFT**  $i$

Press **ENTER**

```
|1+√3i|
2
```

iii Press **MODE** | CPX | 4:angle(

Enter the complex number.

To enter the imaginary symbol  $i$  press number **SHIFT**  $i$

Press **ENTER**

```
angle(1+√3i)
1.047197551
```

## 1.12 Solving equations with complex roots

### Example 15

Solve the equation:

$$2x^3 - 15x^2 + 44x - 39 = 0$$

Press **APPS**. You will see the dialog box as shown on the right.

Choose the App PlySmlt2 and press **ENTER**.

```
APPLICATIONS
1: Finance...
2: CtlgHelp
3: PlySmlt2
```

From the main menu, choose 1: POLY ROOT FINDER and press **EXE**.

```
MAIN MENU
1: POLY ROOT FINDER
2: SIMULT EQN SOLVER
3: ABOUT
4: POLY HELP
5: SIMULT HELP
6: QUIT POLYSMLT
```

▶ Continued on next page

The defaults are to solve an equation of order 2 (a quadratic equation) with real roots. Change the order to 3 and the roots to  $a + bi$ .

Press **F5** NEXT

```

POLY ROOT FINDER MODE
ORDER  1 2 3 4 5 6 7 8 9 10
REAL   0+bi  re^(0i)
DEC     FRAC
NORMAL  SCI ENG
FLOAT   0 1 2 3 4 5 6 7 8 9
RADIAN  DEGREE
(MAIN)  (HELP) (NEXT)

```

Another dialog box opens for you to enter the equation.

The general form of the cubic equation is  $a_3x^3 + a_2x^2 + a_1x + a_0 = 0$  so we enter the coefficients in  $a_3$ ,  $a_2$ ,  $a_1$  and  $a_0$ .

```

a3x^3+...+a1x+a0=0
a3 = 
a2 = 
a1 = 
a0 = 
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)

```

Here  $a_2 = 3$ ,  $a_1 = -4$  and  $a_0 = -2$ . Be sure to use the **(-)** key to enter the negative values. Press **e** after each value.

Press **F5** and the calculator will find the roots of the equation.

```

a3x^3+...+a1x+a0=0
a3 = 2
a2 = -15
a1 = 44
a0 = -39
(MAIN) (MODE) (CLR) (LOAD) (SOLVE)

```

The solutions are,

$x = 3 + 2i$ ,  $x = 3 - 2i$  and  $x = 1.5$ .

```

a3x^3+...+a1x+a0=0
x1 = 3+2i
x2 = 3-2i
x3 = 3/2
(MAIN) (MODE) (COEF) (STD) (F4) (D)

```

## 1.13 Polar form

The GDC displays complex numbers in either Cartesian form ( $z = x + yi$ ) or in Euler's form ( $z = re^{0i}$ ), but not in modulus, argument form – see 1.11 for how to find the modulus and argument of a complex number expressed in Cartesian form.

### Example 16

i Change  $2 + 2i$  to polar form.

ii Change  $3e^{\frac{2\pi i}{3}}$  to Cartesian form.

Complex calculations are entered in the same way as you would enter a real expression.

To enter the imaginary symbol  $i$  press number **SHIFT** **i**

i Enter  $2 + 2i$  and then press **OPTN** | **CPX** | **F3** : **►** Polar

Press **ENTER**

```

2+2i ► Polar
2.828427e-.785398i

```

ii Enter  $3e^{\frac{2\pi i}{3}}$  and then press **OPTN** | **CPX** | **6** : **►** Rect

Press **ENTER**

```

3e^{\frac{2\pi i}{3}} ► Rect
-1.5+2.59807621i

```

► Continued on next page

You can also change the mode that the calculator uses to display complex results in settings.

Press **MODE**

Select REAL,  $a + bi$  for Rectangular or  $re^{\theta i}$  for Polar.

For example, in Polar mode, typing  $2 + 2i$  **ENTER** would result in the number being displayed in polar form without entering **►** Polar.

```

NORMAL  SCI  ENG
FLOAT   0 1 2 3 4 5 6 7 8 9
RADIAN  DEGREE
FUNC    PAR  POL  SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL    a+bi  r e^θi
FULL    HORIZ  G-T
        4NEXT4
  
```

## Exponential functions

### 1.14 Drawing an exponential graph

#### Example 17

Draw the graph of  $y = 3^x + 2$ .

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type  $3^x + 2$  and press **ENTER**.

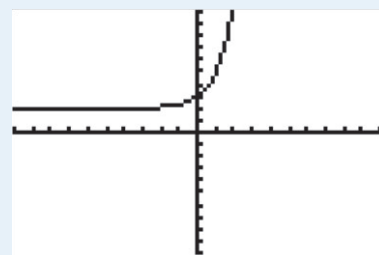
**Note:** Type **3** **^** **X,T,θ,n** **►** to enter  $3^x$ . The **►** returns you to the baseline from the exponent.

Press **ZOOM** **|** **6:ZStandard** to use the default axes which are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .

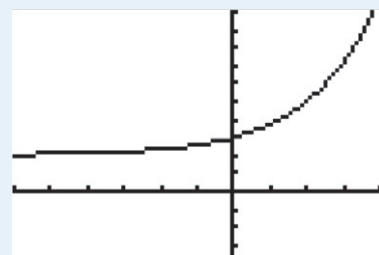
The calculator displays the curve with the default axes.

```

Plot1 Plot2 Plot3
Y1=3^X+2
Y2=
Y3=
Y4=
Y5=
Y6=
  
```



Adjust the window to make the exponential curve fit the screen better.

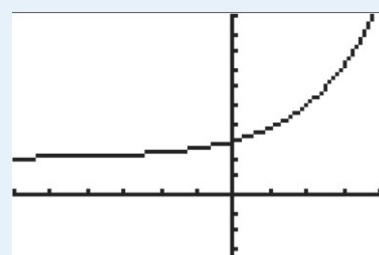


### 1.15 Finding a horizontal asymptote

#### Example 18

Find the horizontal asymptote to the graph of  $y = 3^x + 2$ .

Draw the graph of  $y = 3^x + 2$  (see Example 17).



► Continued on next page

You can look at the graph and a table of the values on the graph by using a split screen.

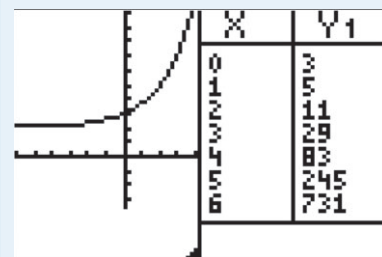
Press **MODE** and select G-T.

Press **GRAPH**.

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
↓NEXT↓
    
```

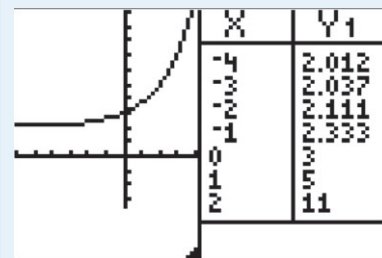
The values of the function are clearly decreasing as  $x \rightarrow 0$ .



Press **2nd** **TABLE** to switch to the table.

Press **▲** to scroll up the table.

The table shows that as the values of  $x$  get smaller,  $Y_1$  approaches 2.

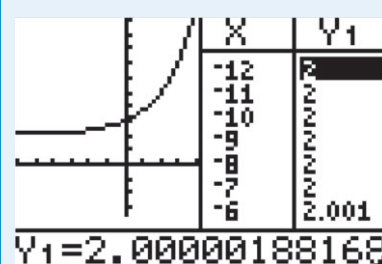


Eventually the value of  $Y_1$  displayed in the table reaches 2.

Press **►** to move to the column containing  $y$ -values. This shows greater precision in the box below the table. You can see, at the bottom of the screen, that the actual value of  $Y_1$  is 2.00000188168...

We can say that  $Y_1 \rightarrow 2$  as  $x \rightarrow -\infty$ .

The line  $x = 2$  is a horizontal asymptote to the curve  $y = 3^x + 2$ .



## Logarithmic functions

### 1.16 Evaluating logarithms

#### Example 19

Evaluate  $\log_{10} 3.95$ ,  $\ln 10.2$  and  $\log_5 2$ .

Press **ALPHA** **F2** | 5:logBASE(to open the log template.

Enter the base and the argument then press **ENTER**.

```

1: abs(
2: Σ(
3: nDeriv(
4: fnInt(
5: logBASE(
[FRAC] [FUNC] [MTRX] [YVAR]
    
```

For natural logarithms it is possible to use the same method, with the base equal to  $e$ , but it is quicker to press **LN**.

Note that the GDC will evaluate logarithms with any base without having to use the change of base formula.

```

log10(3.95)
.5965970956
ln(10.2)
2.32238772
log5(2)
.4306765581
    
```

## 1.17 Finding an inverse function

The inverse of a function can be found by interchanging the  $x$  and  $y$  values. Geometrically this can be done by reflecting points in the line  $y = x$ .

### Example 20

Show that the inverse of the function  $y = 10^x$  is  $y = \log_{10} x$  by reflecting  $y = 10^x$  in the line  $y = x$ .

Draw the line  $y = x$  so that it can be recognised as the axis of reflection.

Press  $\text{Y=}$  to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type  $x$  and press  $\text{ENTER}$ .

Type  $10^x$  and press  $\text{ENTER}$ .

Note: Type  $\text{1}$   $\text{0}$   $\text{^}$   $\text{X,T,}\theta,\text{n}$   $\text{▶}$  to enter  $10^x$ . The  $\text{▶}$  returns you to the baseline from the exponent.

```
Plot1 Plot2 Plot3
Y1=
Y2=10^X
Y3=
Y4=
Y5=
Y6=
```

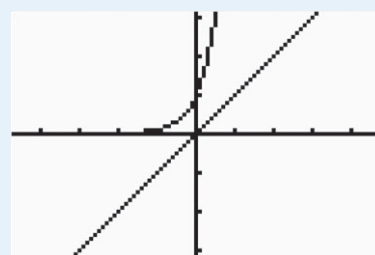
Press  $\text{WINDOW}$  and choose options as shown.

This will set up square axes  $-4.7 \leq x \leq 4.7$  and  $-3.1 \leq y \leq 3.1$ , with the same horizontal and vertical scales.

```
WINDOW
Xmin=-4.7
Xmax=4.7
Xscl=1
Ymin=-3.1
Ymax=3.1
Yscl=1
Xres=1
```

Press  $\text{GRAPH}$ .

The graphs of  $y = x$  and  $y = 10^x$  are displayed.



Press  $\text{2nd}$   $\text{DRAW}$  | 8:DrawInv.

Then press  $\text{ALPHA}$   $\text{F4}$  and choose  $Y_2$ .

Press  $\text{ENTER}$ .

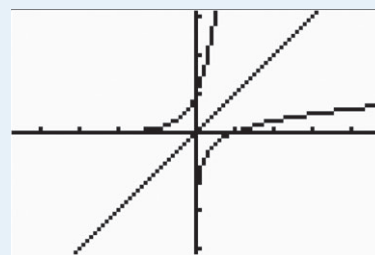
Alternatively press  $\text{LOG}$   $\text{X,T,}\theta,\text{n}$  to enter  $\log(x)$ .  $\text{LOG}$  is a shorter way to enter  $\log_{10}$ .

```
DrawInv Y2
Done
```

Press  $\text{GRAPH}$ .

The graphs are displayed.

The calculator will display the inverse of the function  $y = 10^x$ .



Press  $\text{Y=}$  to display the Y= editor.

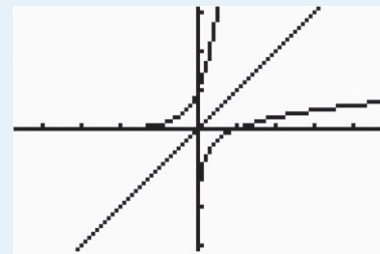
Type  $\log(x)$ .

Press  $\text{LOG}$   $\text{X,T,}\theta,\text{n}$  to enter  $\log(x)$ .  $\text{LOG}$  is a shorter way to enter  $\log_{10}$ .

```
Plot1 Plot2 Plot3
Y1=
Y2=10^X
Y3=log(X)
Y4=
Y5=
Y6=
```

▶ Continued on next page

Press **GRAPH** to display the graphs of  $y = x$ ,  $y = 10^x$  and  $y = \log_{10} x$ .  
The inverse function and the logarithmic function coincide, showing that  $y = \log_{10} x$  is inverse of the function  $y = 10^x$ .



## 1.18 Drawing a logarithmic graph

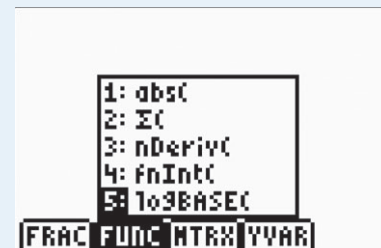
### Example 21

Draw the graph of  $y = 2\log_{10} x + 3$ .

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

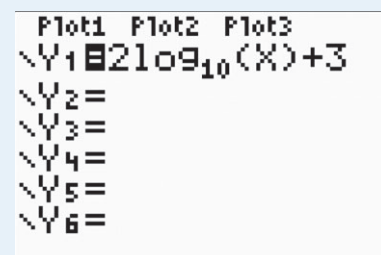
Press **ALPHA** **F2** | 5:logBASE( to open the log template.

Enter the base and the argument then press **ENTER**.

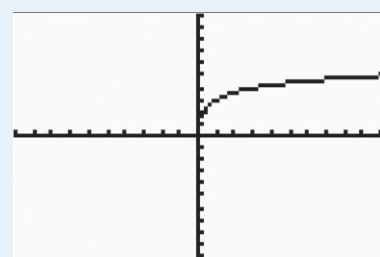


Type  $2\log_{10}(x) + 3$  and press **ENTER**.

Press **ZOOM** 6:XStandard so that the calculator displays the curve with the default axes.

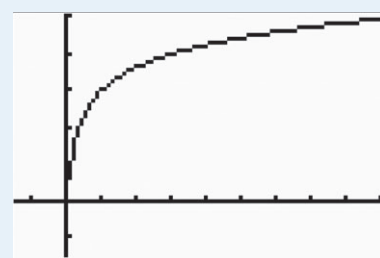


The calculator displays the curve with the default axes.



Change the axes to make the logarithmic curve fit the screen better.

For help with changing axes, see your GDC manual.





## Trigonometric functions

### 1.19 Degrees and radians

Work in trigonometry will be carried out either in degrees or radians. It is important, therefore, to be able to check which mode the calculator is in and to be able to switch back and forth

#### Example 22

Change angle settings from radians to degrees and from degrees to radians.

Press **MODE**.

Select either RADIAN or DEGREE using the **▶** **◀** **▲** **▼** keys.

Press **ENTER**.

Press **2nd** **QUIT**.

```

NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi re^θi
FULL HORIZ G-T
↓NEXT↓
  
```

### 1.20 Drawing a trigonometric graph

#### Example 23

Draw the graph of  $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$ .

Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

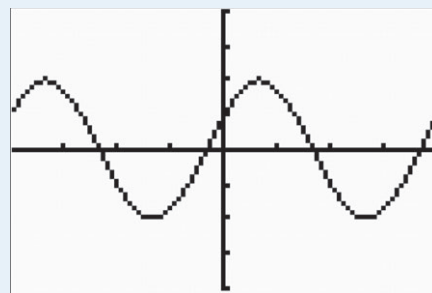
Type  $y = 2\sin\left(x + \frac{\pi}{6}\right) + 1$  and press **ENTER**.

```

Plot1 Plot2 Plot3
Y1=2sin(X+π/6)
Y2=
Y3=
Y4=
Y5=
Y6=
  
```

Press **ZOOM** **7:ZTrig**.

The default axes are  $-6.15 \leq x \leq 6.15$  and  $-4 \leq y \leq 4$ .



## More complicated functions

### 1.21 Solving a combined quadratic and exponential equation

#### Example 24

Solve the equation  $x^2 - 2x + 3 = 3 \cdot 2^{-x} + 4$

To solve the equation, find the point of intersection between the quadratic function  $y_1 = x^2 - 2x + 3$  and the exponential function  $y_2 = 3 \times 2^{-x} + 3$ .

Press  $\boxed{Y=}$  to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

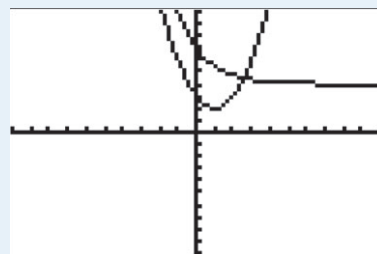
Type  $x^2 - 2x + 3$  in  $Y_1$  and press  $\boxed{\text{ENTER}}$ . Then type  $3 \times 2^{-x} + 4$  in  $Y_2$  and press  $\boxed{\text{ENTER}}$ .

(**Note:** Type  $\boxed{2} \boxed{\wedge} \boxed{(-)} \boxed{x,T,\theta,n} \boxed{\blacktriangleright}$  to enter  $2^{-x}$ . The  $\boxed{\blacktriangleright}$  returns you to the baseline from the exponent.)

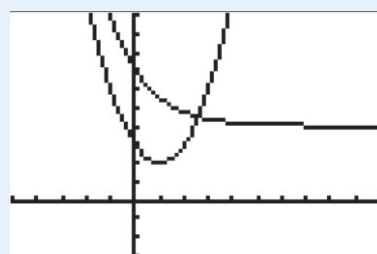
```
Plot1 Plot2 Plot3
Y1=X^2-2X+3
Y2=3*2^-X+4
Y3=
Y4=
Y5=
Y6=
```

Press  $\boxed{\text{ZOOM}} \boxed{6} \boxed{Z}$  Standard to use the default axes which are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .

The calculator displays the curves with the default axes.



Adjust the window to make the quadratic curve fit the screen better.



Press  $\boxed{2\text{nd}} \boxed{\text{CALC}} \boxed{5}$ :intersect.

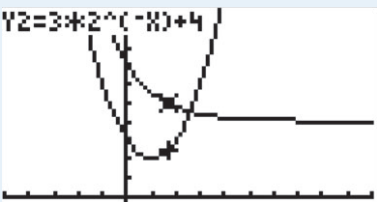
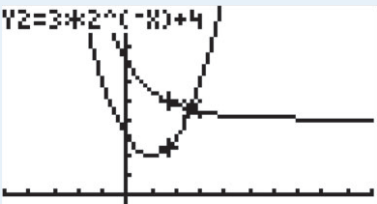
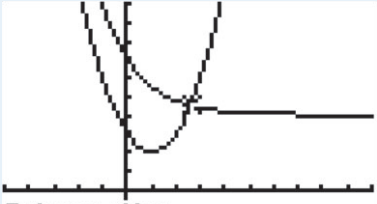
Press  $\boxed{\text{ENTER}}$ .

```
CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
```

Press  $\boxed{\text{ENTER}}$  to select the first curve.

```
Y1=X^2-2X+3
First curve?
X=1.7021277 Y=2.4929833
```

► Continued on next page

Press <b>ENTER</b> to select the second curve.	 <p>Second curve? X=1.7021277 Y=4.9219976</p>
Select a point close to the intersection using the <b>◀</b> and <b>▶</b> keys and press <b>ENTER</b> .	 <p>Guess? X=2.6595745 Y=4.4747987</p>
<p>The calculator displays the intersection of the two straight lines at the point (2.58, 4.50).</p> <p>The solutions are <math>x = 2.58</math> and <math>y = 4.50</math>.</p>	 <p>Intersection X=2.5815169 Y=4.5011956</p>

## Sequences and series

### 1.22 Summation of a series

#### Example 25

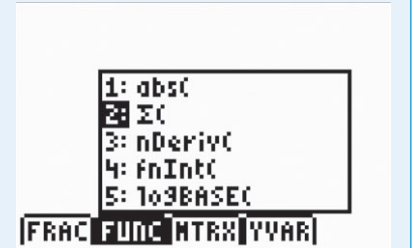
Find the sum of the first 20 terms of the arithmetic sequence 4, 7, 10, 13, ...

The  $k$ th term of an arithmetic sequence is  $u_k = u_1 + (k - 1)d$

In this example,  $u_1 = 4$ ,  $d = 3$  and  $n = 20$ .

$$s_n = \sum_{k=1}^{20} 4 + (k - 1)3.$$

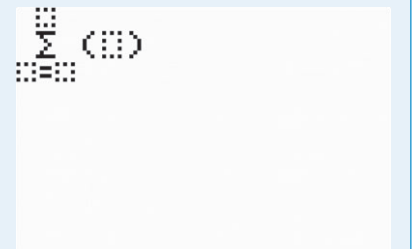
Press **ALPHA** **F2** and select 2:Σ(



The template matches the written Sigma formula.

Enter the variables, values and the function as they are written.

Use the **▶** **◀** **▲** **▼** keys to move around the template.



▶ Continued on next page

The sum of the terms of the sequence is 650.

$$\sum_{k=1}^{20} (4 + (k-1)3) = 650$$

### Example 26

Find the sum of the first 12 terms of the geometric sequence  $3, -1, \frac{1}{3}, -\frac{1}{9}, \dots$

The  $k$ th term of a geometric sequence is  $u_k = u_1 \cdot r^{k-1}$

In the example  $u_1 = 3$ ,  $r = -\frac{1}{3}$  and  $n = 12$ .

$$s_n = \sum_{k=1}^{12} 3 \cdot \left(-\frac{1}{3}\right)^{k-1}$$

Press **ALPHA** **F2** and select 2:Σ(

```

1: abs(
2: Σ(
3: nDeriv(
4: fnInt(
5: logBASE(
[FRAC] [FUNC] [MTRX] [YVAR]

```

The template matches the written Sigma formula.

Enter the variables, values and the function as they are written.

Use the **▶** **◀** **▲** **▼** keys to move around the template.

```

Σ(
  ( )
  :
  :
  :

```

The sum of the terms of the sequence is 2.25, to 3 significant figures.

$$\sum_{k=1}^{12} \left( 3 \left( -\frac{1}{3} \right)^{k-1} \right) = 2.249995766$$

### Example 27

How many terms of the series  $2 + 1\frac{1}{3} + \frac{8}{9} + \frac{16}{27} + \dots$  are needed before their sum exceeds 5.5?

In the example  $u_1 = 2$ ,  $r = \frac{2}{3}$  and  $n$  is to be found.

$$s_n = \sum_{k=1}^n 2 \cdot \left(\frac{2}{3}\right)^{k-1}$$

Press !

Press **ALPHA** **F2** and select 2:Σ(

```





Plot1 Plot2 Plot3
\Y1=
\Y2=1: abs(
\Y3=2: Σ(
\Y4=3: nDeriv(
\Y5=4: fnInt(
\Y6=5: logBASE(
[FRAC] [FUNC] [MTRX] [YVAR]

```

▶ Continued on next page

The template matches the written Sigma formula.

Enter the variables, values and the function as they are written.

Use the     keys to move around the template.




In place of the value for  $n$  type  $x$ .

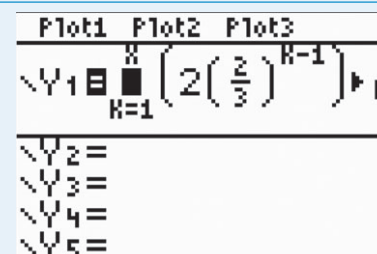
Type the Sigma formula for the series.

Press **ENTER**

This stores the Sigma formula as the function, Y1.

Instead of looking at the function as a graph, you will look at it in a table.

The TI-84 Plus displays values in the table as exact, so in order to see the values of Y1 as decimal fractions press **MODE** | MATH | 2:  Dec



Press **2nd** TABLE

You will see a list of the sums of the series for different values of  $x$ .

The table shows that when  $n=7$ ,  $S_n > 5.5$  as required.

X	Y1	
1	2	
2	3.3333	
3	4.2222	
4	4.8148	
5	5.2099	
6	5.4733	
7	5.6488	

Press + for  $\Delta$ Tbl

## Modelling

### 1.23 Using sinusoidal regression

The notation  $\sin^2 x$ ,  $\cos^2 x$ ,  $\tan^2 x$ , ... is a mathematical convention that has little algebraic meaning. To enter these functions on the GDC, you should enter  $(\sin(x))^2$ , etc. However, the calculator will conveniently interpret  $\sin(x)^2$  as  $(\sin(x))^2$ .

#### Example 28


It is known that the following data can be modelled using a sine curve:


x	0	1	2	3	4	5	6	7
y	6.9	9.4	7.9	6.7	9.2	8.3	6.5	8.9

Use sine regression to find a function to model this data.

Press **STAT** | 1:Edit and press **F3**.

Type the  $x$ -values in the first column (L1) and the  $y$ -values in the second column (L2).

Press **ENTER** or  after each number to move down to the next cell.

Press  to move to the next column.

You can use columns from L1 to L6 to enter the lists.

L1	L2	L3	1
0	6.9	-----	
1	9.4		
2	7.9		
3	6.7		
4	9.2		
5	8.3		
6	6.5		

L1()=0

► Continued on next page

Press **2nd** **STAT PLOT** and **eto** select Plot1.

Select **On**, choose the scatter diagram option, XList as L1 and Ylist as L2.

You can choose one of the three types of mark.

```

STAT PLOTS
1:Plot1...Off
  [X] L1 L2
2:Plot2...Off
  [X] L1 L2
3:Plot3...Off
  [X] L1 L2
4:PlotsOff

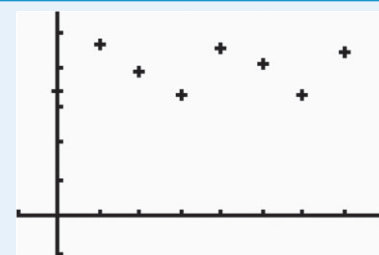
Plot1 Plot2 Plot3
Off Off Off
Type: [Scatter] [Line] [Line]
      [On] [Off] [Off]
Xlist:L1
Ylist:L2
Mark: [ ] [ ] [ ]

```

Press **ZOOM** **9:ZoomStat**.

Adjust your window settings to show your data and the  $x$ - and  $y$ -axes.

You now have a scatter plot of  $x$  against  $y$ .



Press **2nd** **↑** to return to the Home screen.

Press **STAT** **CALC** | **C:SinReg**.

Press **2nd** **L1** , **2nd** **α**, **ALPHA** **F4** choose  $Y_1$  and press **F3**.

Press **F3** again.

```
SinReg L1,L2,Y1
```

On screen, you will see the result of the sinusoidal regression.

The equation is in the form  $y = a\sin(bx + c) + d$  and you will see the values of  $a$ ,  $b$ ,  $c$  and  $d$  displayed separately.

The equation of the sinusoidal regression line is

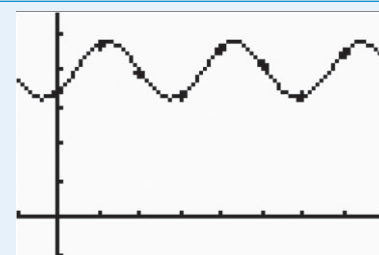
$$y = 1.51\sin(2.00x - 0.80) + 7.99$$

```

SinReg
y=a*sin(bx+c)+d
a=1.506000561
b=2.002900961
c=-.7998734807
d=7.991078656

```

Press **GRAPH** to return to the Graphs page.



Press **▸**.

The regression line is now shown in  $Y_1$ . You can see the full equation if you scroll to the right.

```

2nd Y1 Plot2 Plot3
Y1=1.506000561sin(2.002900961x-.7998734807)+7.991078656
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=

```



## 1.24 Drawing a piecewise function

### Example 29

Draw the function  $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ \frac{1}{2}(x - 3), & x \geq 3 \end{cases}$

Press  $\text{Y=}$  to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type  $(x^2 - 4x + 3)(x < 3)$  in  $Y_1$  and press  $\text{ENTER}$ .

Type  $\left(\frac{1}{2}(x - 3)\right)(x \geq 3)$  in  $Y_2$  and press  $\text{ENTER}$ .

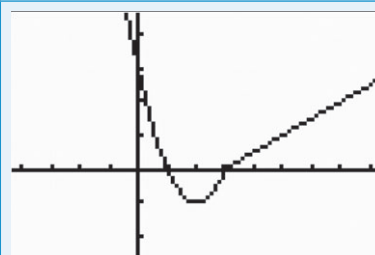
```
P1ot1 P1ot2 P1ot3
\Y1=(X^2-4X+3)(X<3)
\Y2=1/2(X-3)(X≥3)
\Y3=
\Y4=
\Y5=
```

To enter the inequalities for the domain use  $\text{2nd}$  TEST.

Press  $\text{GRAPH}$

Choose suitable axes to display the curves.

The piecewise function is displayed.



## 2 Differential calculus

### Finding gradients, tangents and maximum and minimum points

#### 2.1 Finding the gradient at a point

### Example 30

Find the gradient of the cubic function  $y = x^3 - 2x^2 - 6x + 5$  at the point where  $x = 1.5$ .

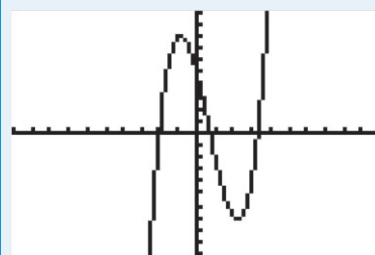
Press  $\text{Y=}$  to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type  $y = x^3 - 2x^2 - 6x + 5$  and press  $\text{ENTER}$ .

Note: Type  $X, T, \theta, n$   $\wedge$   $3$   $\rightarrow$  to enter  $x^3$ . The  $\rightarrow$  returns you to the baseline from the exponent.

```
P1ot1 P1ot2 P1ot3
\Y1=X^3-2X^2-6X+5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

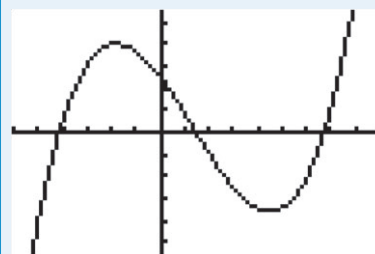
Press  $\text{ZOOM}$  | 6:ZStandard to use the default axes which are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .



► Continued on next page

Adjust the window to make the cubic curve fit the screen better.

For help with changing axes, see your GDC manual.



Press **2nd** **CALC** | 6:  $dy/dx$ .

Press **ENTER**.

Press **1** **.** **5** **ENTER**.

```

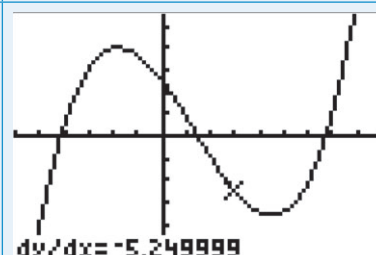
MATH>
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx

```

The calculator displays the gradient of the curve at the point where  $x = 1.5$ .

The gradient is  $-5.25$ .

In this example the value of  $dy/dx$  is not exactly  $-5.25$ . This is due to the way the calculator finds the point gradient. You should ignore small errors like this when you write down the coordinates of a gradient at a point.

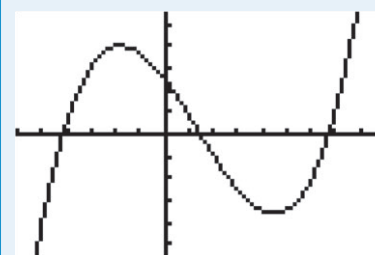


## 2.2 Drawing a tangent to a curve

### Example 31

Draw a tangent to the curve  $y = x^3 - 2x^2 - 6x + 5$  where  $x = -0.5$ .

First draw the graph of  $y = x^3 - 2x^2 - 6x + 5$  (see Example 30).



Press **2nd** **DRAW**.

Choose 5:Tangent.

Press **ENTER**.

```

2ND DRAW POINTS STO
1:ClrDraw
2:Line(
3:Horizontal
4:Vertical
5:Tangent(
6:DrawF
7:↓Shade(

```

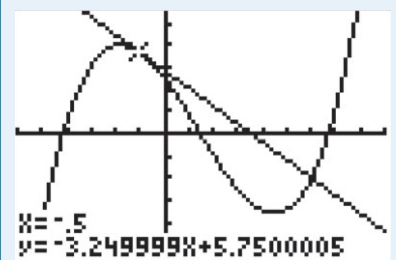
► Continued on next page

Press  $(-)$  0 . 5  $\text{ENTER}$ .

The equation of the tangent is

$$y = -3.25x + 5.75$$

In this example the values  $-3.25$  and  $5.75$  are not shown as being exact. This is due to the way the calculator finds the values. You should ignore small errors like this when you write down the equation of a tangent.

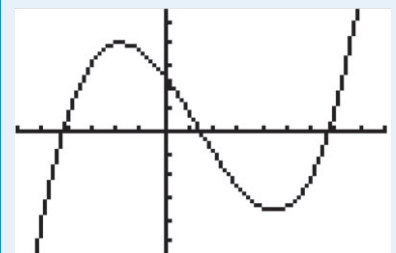


## 2.3 Finding maximum and minimum points

### Example 32

Find the local maximum and local minimum points on the cubic curve.

First draw the graph of  $y = x^3 - 2x^2 - 6x + 5$  (see Example 30).



Press  $2\text{nd}$   $\text{CALC}$  | 3:minimum.

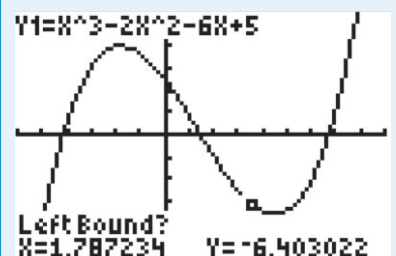
Press  $\text{ENTER}$ .

**CALCULATE**  
1:value  
2:zero  
3:minimum  
4:maximum  
5:intersect  
6:dy/dx  
7:ff(x)dx

To find the minimum point you need to give the left and right bounds of a region that includes it.

The calculator shows a point and asks you to set the left bound. Move the point using the  $\leftarrow$  and  $\rightarrow$  keys to choose a position to the left of the minimum.

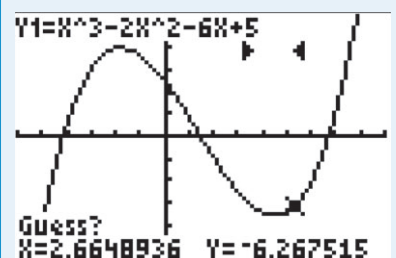
Press  $\text{ENTER}$ .



The calculator shows another point and asks you to set the right bound.

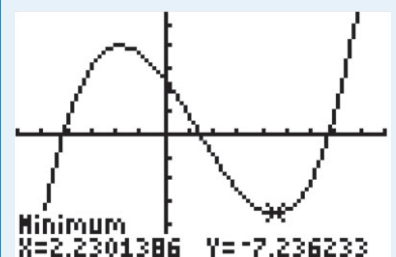
Move the point using the  $\leftarrow$  and  $\rightarrow$  keys so that the region between the left and right bounds contains the minimum.

When the region contains the minimum press  $\text{ENTER}$ .



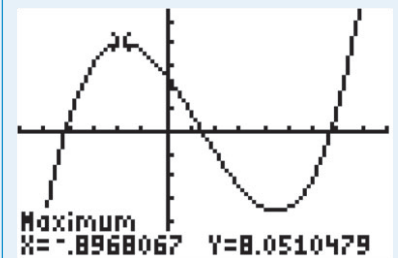
Press  $\text{ENTER}$  again to supply a guess for the value of the minimum.

The calculator displays the local minimum at the point  $(2.23, -7.24)$ .



► Continued on next page

Press **2nd** **CALC** | 3:maximum **ENTER**. To find the local maximum point on the curve in exactly the same way.  
The maximum point is  $(-0.897, 8.05)$ .



## 2.4 Finding a numerical derivative

Using the calculator it is possible to find the numerical value of any derivative for any value of  $x$ . The calculator will not, however, differentiate a function algebraically. This is equivalent to finding the gradient at a point graphically (see Section 2.1 example 30).

### Example 33

If  $y = \frac{x+3}{x}$ , evaluate  $\left. \frac{dy}{dx} \right|_{x=2}$

Press **ALPHA** **F2**.  
Choose 3: nDeriv( to choose the derivative template.

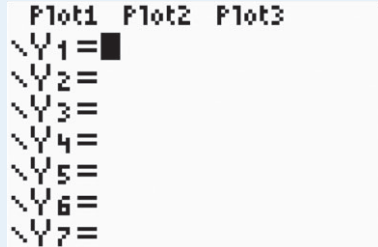
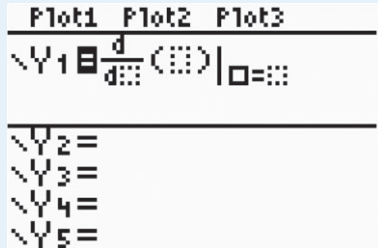
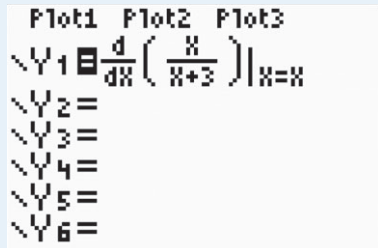
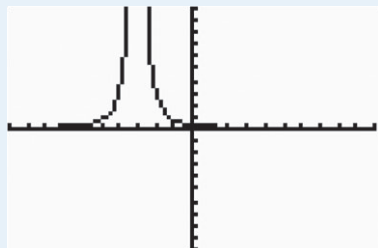
Enter  $x$  and the function in the template. Enter the value 2.  
Press **ENTER**.

The calculator shows that the value of the first derivative of  $y = \frac{x+3}{x}$  is  $-0.75$  when  $x = 2$ .

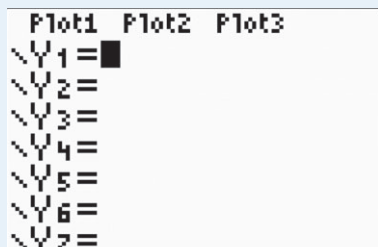
## 2.5 Graphing a numerical derivative

Although the calculator can only evaluate a numerical derivative at a point, it will graph the gradient function for all values of  $x$ .

### Example 34

If $y = \frac{x}{x+3}$ , draw the graph of $\frac{dy}{dx}$ .	
Press $\text{Y=}$ to display the Y= editor. The default graph type is Function, so the form Y= is displayed.	
Press $\text{ALPHA}$ $\text{F2}$ . Choose 3: nDeriv (to choose the derivative template).	
In the template enter $x$ , the function $\frac{x}{x+3}$ and the value $x$ . Press $\text{ENTER}$ .	
Press $\text{ZOOM}$ $\text{6:ZStandard}$ . The calculator displays the graph of the numerical derivative function of $y = \frac{x}{x+3}$ .	

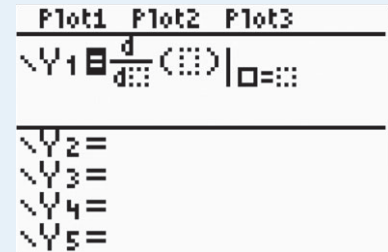
### Example 35

Find the values of $x$ on the curve $y = \frac{x^3}{3} + x^2 - 5x + 1$ where the gradient is 3.	
Press $\text{Y=}$ to display the Y= editor. The default graph type is Function, so the form Y= is displayed.	

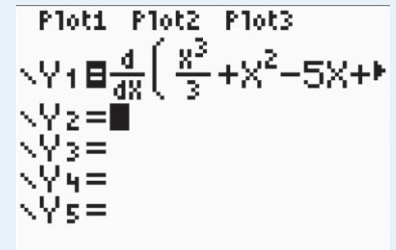
► Continued on next page

Press **ALPHA** **F2**.

Choose 3: nDeriv( to choose the derivative template.

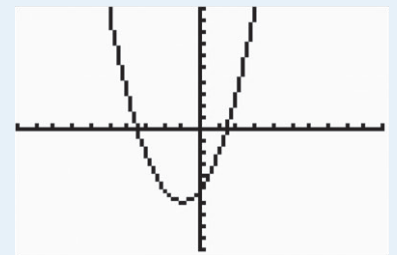


In the template enter  $x$ , the function  $\frac{x^3}{3} + x^2 - 5x + 1$  and the value  $x$ .  
Press **ENTER**.



Press **ZOOM** 6:ZStandard.

The calculator displays the graph of the numerical derivative function of  $y = \frac{x^3}{3} + x^2 - 5x + 1$ .

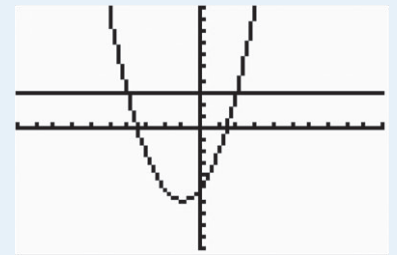


Press **Y=** to display the Y= editor.

Enter the function  $Y_2 = 3$ .

Press **GRAPH**.

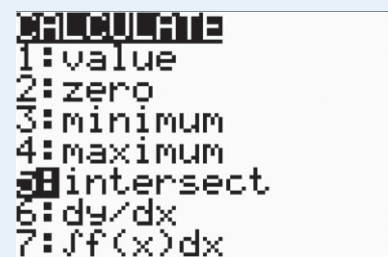
The calculator now displays the curve and the line  $y = 3$ .



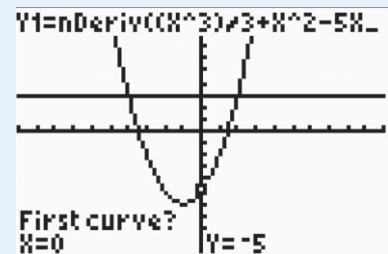
To find the points of intersection between the curve and the line.

Press **2nd** **CALC** | 5:intersect.

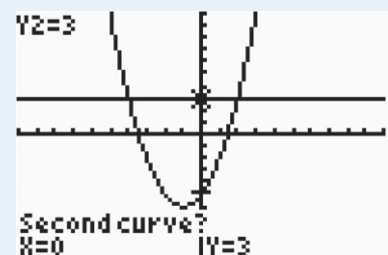
Press **ENTER**.



Press **ENTER** to select the first curve.



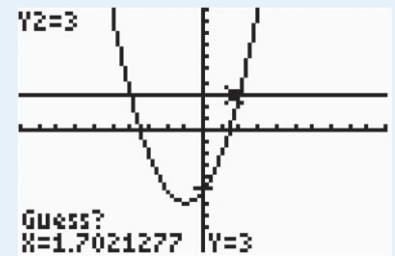
Press **ENTER** to select the second curve.



▶ Continued on next page

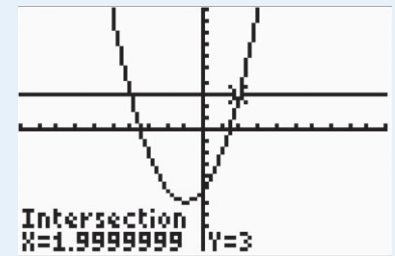
Select a point close to the intersection using the  $\leftarrow$  and  $\rightarrow$  keys and press  $\text{ENTER}$ .

Repeat for the second point of intersection.



The curve has gradient 3 when  $x = -4$  and  $x = 2$

In this example the value of  $x$  is not exactly 2. This is due to the way the calculator finds the point. You should ignore small errors like this when you write down the coordinates of a gradient at a point.



## 2.6 Using the second derivative

There is no second derivative function on the TI-84 Plus.

# 3 Integral calculus

The calculator can find the values of definite integrals either on a calculator page or graphically. The calculator method is quicker, but the graphical method is clearer and shows discontinuities, negative areas and other anomalies that can arise.

## 3.1 Finding the value of a definite integral

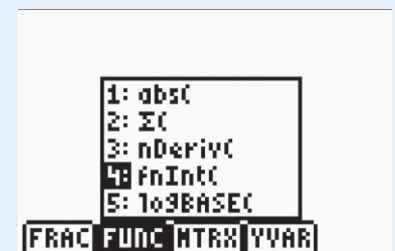
### Example 36

Evaluate  $\int_2^8 \left( x - \frac{3}{\sqrt{x}} \right) dx$ .

Press  $\text{ALPHA}$   $\text{F2}$ .

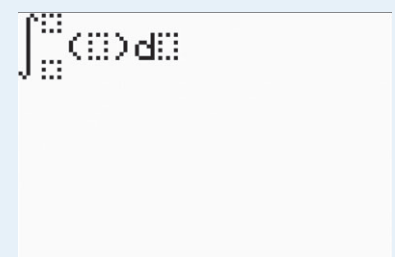
Choose 4: fnInt( to choose the integral template.

In this example you will also use templates to enter the rational function and the square root.



Enter the upper and lower limits, the function and  $x$  in the template.

Press  $\text{ENTER}$ .



► Continued on next page



The value of the integral is 21.5 (to 3 sf).

$$\int_2^8 \left( x - \frac{3}{\sqrt{x}} \right) dx$$

21.51471863

## 3.2 Finding the area under a curve

### Example 37

Find the area bounded by the curve  $y = 3x^2 - 5$ , the  $x$ -axis and the lines  $x = -1$  and  $x = 1$ .

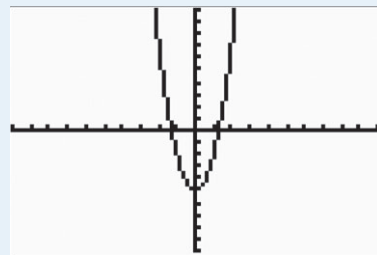
Press **Y=** to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Type  $y = 3x^2 - 5$  and press **ENTER**.

Plot1 Plot2 Plot3  
 $Y_1 = 3X^2 - 5$   
 $Y_2 =$   
 $Y_3 =$   
 $Y_4 =$   
 $Y_5 =$   
 $Y_6 =$

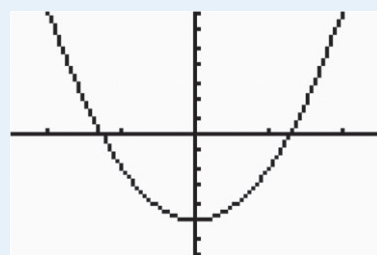
Press **ZOOM** 6:ZStandard.

The default axes are  $-10 \leq x \leq 10$  and  $-10 \leq y \leq 10$ .



Adjust the window settings to view the curve better.

For help with changing axes, see your GDC manual.

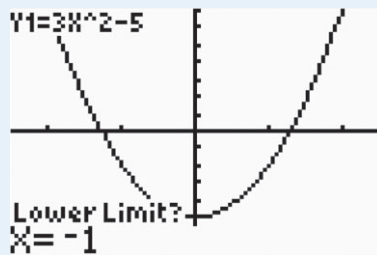


Press **2nd** **CALC** 7:∫f(x)dx.

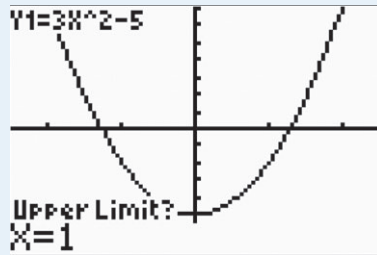
The calculator prompts you to enter the lower limit for the integral.

Type -1 and press **ENTER**.

Be sure to use the **(-)** key.



Type 1 and press **ENTER**.

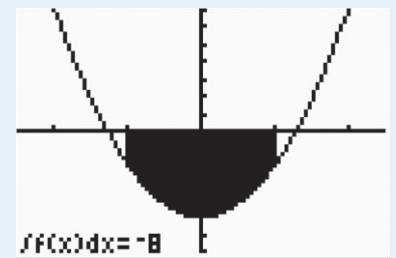


▶ Continued on next page

The area found is shaded and the value of the integral ( $-8$ ) is shown on the screen.

The required area is 8.

Since the area lies below the x-axis in this case, the integral is negative.



## 4 Vectors

### Scalar product

#### 4.1 Calculating a scalar product

##### Example 38

Evaluate the scalar products:

**a**  $\begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

**b**  $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

**a** Press **2nd** LIST | MATH | 5:sum(.

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:Prod(
7:stdDev(
```

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.

Multiply the two vector lists together.

```
4m({1,3}*{-3,4})
```

Close the bracket and press **ENTER**.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = 9$$

```
sum({1,3}*{-3,4})
9
```

**b** Press **2nd** LIST | MATH | 5:sum(.

```
NAMES OPS MATH
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:Prod(
7:stdDev(
```

There is no scalar product function on the TI-84 Plus, but you can find the result by multiplying the vectors as lists and then finding the sum of the terms in the list.

► Continued on next page

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.

Multiply the two vector lists together.

4. -1, 4} \* {3, 2, -1}▶

Close the bracket and press **ENTER**.

$$\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = -3$$

sum({1, -1, 4} \* {3▶  
-3

## 4.2 Calculating the angle between two vectors

The angle  $\theta$  between two vectors  $\vec{a}$  and  $\vec{b}$ , can be calculated using the formula

$$\theta = \arcsin\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}\right)$$

### Example 39

Calculate the angle between  $2\vec{i} + 3\vec{j}$  and  $3\vec{i} - \vec{j}$

Press **MODE**.

Select either RADIAN or DEGREE (according to the units you need your answer in) using the **▶** **◀** **▲** **▼** keys.

Press **ENTER**.

Press **2nd** **QUIT**.

```
NORMAL SCI ENG
FLOAT 0 1 2 3 4 5 6 7 8 9
RADIAN DEGREE
FUNC PAR POL SEQ
CONNECTED DOT
SEQUENTIAL SIMUL
REAL a+bi Re^θi
FULL HORIZ G-T
4NEXT4
```

Press **2nd** **DISTR**.

cos<sup>-1</sup>(

Press **ALPHA** **F1** and select the fraction template 1:n/d

cos<sup>-1</sup>(

```
1: n/d
2: Un/d
3: n/d d Un/d
4: F4 D
FRAC FUNC MTRX YVAR
```

▶ Continued on next page

Press **2nd** LIST | MATH | 5:sum(.


```
NAMES OPS 10:11:14
1:min(
2:max(
3:mean(
4:median(
5:sum(
6:prod(
7:stdDev(
```

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.

Multiply the two vector lists together.

To calculate the magnitudes of the vectors use the formula

$$|a\vec{i} + b\vec{j}| = \sqrt{a^2 + b^2}$$

Use the  key to exit the templates before entering the final bracket.

$$\left( \frac{\text{sum}(\langle 2, 3 \rangle * \langle 3, -1 \rangle)}{\sqrt{2^2 + 3^2} * \sqrt{3^2 + (-1)^2}} \right)$$
$$\cos^{-1}\left(\frac{\text{sum}(\langle 2,3 \rangle * \langle 3, \right.$$
  

$$\frac{\sqrt{2^2+3^2} * \sqrt{3^2+(-1)^2}}{74.7448813}$$

## Vector product

### 4.3 Calculating a vector product

The TI-84 Plus does not have the ability to perform cross products of vectors.

## 5 Statistics and probability

You can use your GDC to draw charts to represent data and to calculate basic statistics such as mean, median, etc. Before you do this you need to enter the data in a list.

## Entering data

There are two ways of entering data: as a list or as a frequency table.

## 5.1 Entering lists of data

### Example 40

Enter the data in the list: 1, 1, 3, 9, 2.

---

Press **STAT** 1: Edit and press **ENTER**.

Type the numbers in the first column (L1).

Press **ENTER** or **▼** after each number to move down to the next cell.

L1 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from L1 to L6 to enter the list.

Press **STAT** 1: Edit and press **ENTER**.  
Type the numbers in the first column (L1).  
Press **ENTER** or  $\blacktriangledown$  after each number to move down to the next cell.  
L1 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from L1 to L6 to enter the list.

```

EDIT CALC TESTS
1:Edit...
2:SortA(
3:SortD(
4:ClrList
5:SetUpEditor

```

L1	L2	L3
1 1 w g v e	-----	-----
<b>L1(6)=</b>		

## 5.2 Entering data from a frequency table

### Example 41

Enter the data in the table:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Press **STAT** | 1:Edit and press **ENTER**.

Type the numbers in the first column (L1) and the frequencies in the second column (L2).

Press **ENTER** or **▼** after each number to move down to the next cell.

Press **▶** to move to the next column.

L1 and L2 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from L1 to L6 to enter the lists.

**EDIT** CALC TESTS  
1:Edit...  
2:SortA(  
3:SortD(  
4:ClrList  
5:SetUpEditor

L1	L2	L3	2
1	3		
2	4		
3	6		
4	5		
5	2		

## Drawing charts

Charts can be drawn from a list or from a frequency table.

## 5.3 Drawing a frequency histogram from a list

### Example 42

Draw a frequency histogram for this data: 1, 1, 3, 9, 2.

Enter the data in L1 (see Example 40).

Press **2nd** **STAT PLOT** and **ENTER** to select Plot1.

Select On, choose the histogram option and leave Xlist as L1 and Freq as 1.

**STAT PLOTS**  
1:Plot1...Off  
2:Plot2...Off  
3:Plot3...Off  
4↓PlotsOff

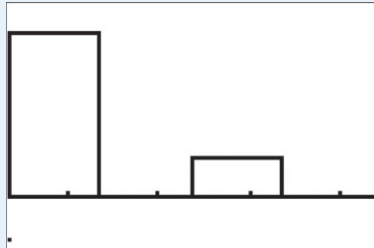
Plot1 Plot2 Plot3  
On Off Off  
Type: ▽ ▽ ▽  
Xlist: L1  
Freq: 1

Press **ZOOM** | 9:Stat.

The automatic scales do not usually give the best display of the histogram. You will need to change the default values.

You may need to delete any function graphs. **Y=**

**ZOOM** MEMORY  
3↑Zoom Out  
4:ZDecimal  
5:ZSquare  
6:ZStandard  
7:ZTrig  
8:ZInteger  
9↓ZoomStat



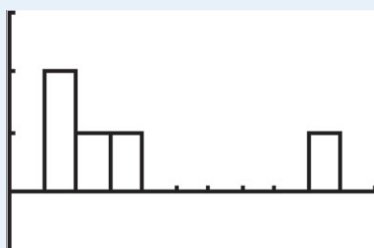
Press **WINDOW** and choose options as shown.

Xmin and Xmax should include the range of the data.

Ymin and Ymax should include the maximum frequency and should go below zero.

Xscl will define the width of the bars.

**WINDOW**  
Xmin=0  
Xmax=11  
Xscl=1  
Ymin=-1  
Ymax=3  
Yscl=1  
↓Xres=1

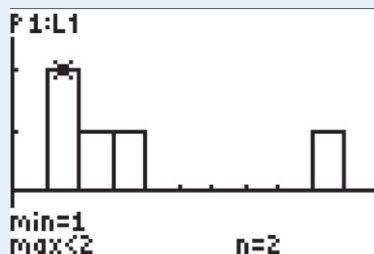


▶ Continued on next page

Press **TRACE**.

Use the **▶** key to move to each of the bars and display their value and frequency.

You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.



## 5.4 Drawing a frequency histogram from a frequency table

### Example 43

Draw a frequency histogram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in L1 and L2 (see Example 41). Press **2nd** **STAT PLOT** and **ENTER** to select Plot 1. Select On, choose the histogram option and leave XList as L1 and Freq as L2.

```

STAT PLOTS
1:Plot1...Off
  L1 L2
2:Plot2...Off
  L1 L2
3:Plot3...Off
  L1 L2
4↓PlotsOff
  
```

```

Plot1 Plot2 Plot3
On Off Off
Type: L1 L2 L3
Xlist:L1
Freq:L2
  
```

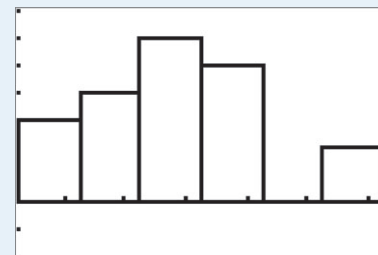
Press **ZOOM** | 9:Stat.

The automatic scales do not usually give the best display of the histogram. You will need to change the default values.

You may need to delete any function graphs. **Y=**

```

ZOOM MEMORY
3↑Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9↓ZoomStat
  
```



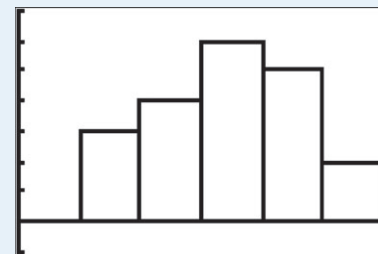
Press **WINDOW** and choose options as shown. Xmin and Xmax should include the range of the data.

Ymin and Ymax should include the maximum frequency and should go below zero.

Xscl will define the width of the bars.

```

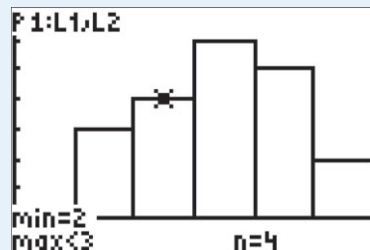
WINDOW
Xmin=0
Xmax=6
Xscl=1
Ymin=-1
Ymax=7
Yscl=1
↓Xres=1
  
```



Press **TRACE**.

Use the **▶** key to move to each of the bars and display their value and frequency.

You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.



## 5.5 Drawing a box and whisker diagram from a list

### Example 44

Draw a box and whisker diagram for this data:

1, 1, 3, 9, 2.

Enter the data in L1 (see Example 40).

Press **2nd** **STAT PLOT** and **ENTER** to select Plot 1.

Select On, choose the box and whisker option and leave XList as L1 and Freq as 1.

```

STAT PLOTS
1:Plot1...Off
  [ ] L1  L2  [ ]
2:Plot2...Off
  [ ] L1  L2  [ ]
3:Plot3...Off
  [ ] L1  L2  [ ]
4↓PlotsOff
  
```

```

Plot1 Plot2 Plot3
On Off Off
Type: [ ] [ ] [ ]
Xlist:L1
Freq:1
  
```

Press **ZOOM** | 9:Stat.

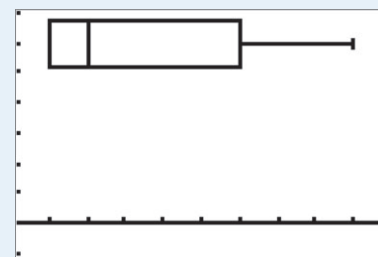
The automatic scales do not usually give the best display of the box and whisker diagram. You will need to change the default values.

You may need to delete any function graphs.

**Y=**

```

ZOOM MEMORY
3↑Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9↓ZoomStat
  
```



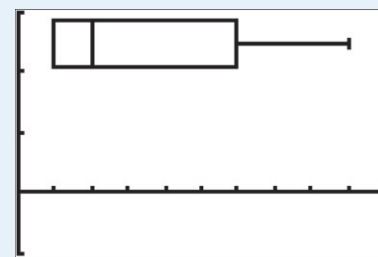
Press **WINDOW** and choose options as shown.

Xmin and Xmax should include the range of the data.

Ymin and Ymax do not affect the way in which the diagram is displayed.

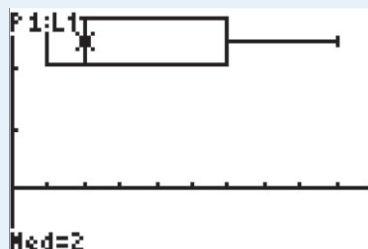
```

WINDOW
Xmin=0
Xmax=10
Xscl=1
Ymin=-1
Ymax=3
Yscl=1
↓Xres=1
  
```



Press **TRACE**.

Use the **→** key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.



## 5.6 Drawing a box and whisker diagram from a frequency table


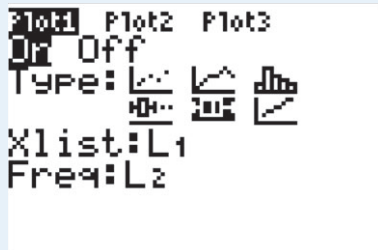
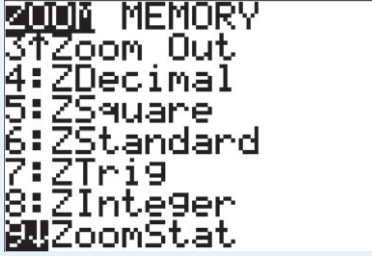
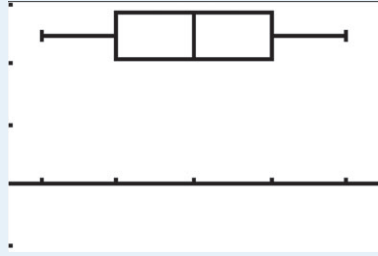
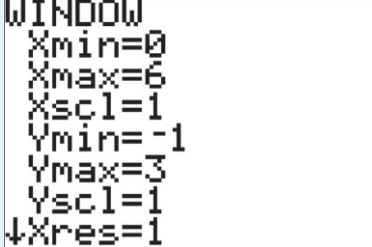
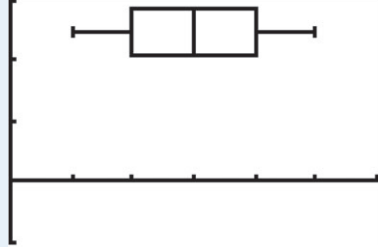
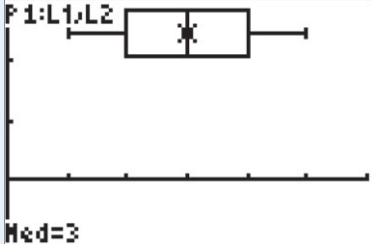
### Example 45

Draw a box and whisker diagram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

► Continued on next page



<p>Enter the data in L1 and L2 (see Example 41).</p> <p>Press <b>2nd</b> <b>STAT PLOT</b> and <b>ENTER</b> to select Plot 1. Select On, choose the box and whisker diagram option and leave Xlist as L1 and Freq as L2.</p>		
<p>Press <b>ZOOM</b>   9:Stat.</p> <p>The automatic scales do not usually give the best display of the box and whisker diagram. You will need to change the default values.</p> <div data-bbox="446 459 651 622" style="border: 1px solid orange; padding: 5px; margin-top: 10px;"> <p>You may need to delete any function graphs. <b>Y=</b></p> </div>		
<p>Press <b>WINDOW</b> and choose options as shown. Xmin and Xmax should include the range of the data. Ymin and Ymax do not affect the way in which the diagram is displayed.</p>		
<p>Press <b>TRACE</b>.</p> <p>Use the <b>▶</b> key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.</p>		

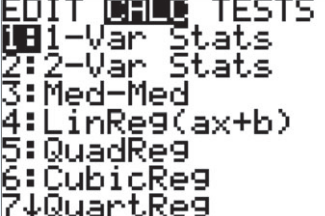

## Calculating statistics

You can calculate statistics such as mean, median, etc. from a list, or from a frequency table.

### 5.7 Calculating statistics from a list

#### Example 46

Calculate the summary statistics for this data: 1, 1, 3, 9, 2

<p>Enter the data in L1 (see Example 40).</p> <p>Press <b>STAT</b>   <b>CALC</b>   1:1-Var Stats.</p> <p>Type <b>2nd</b> <b>L1</b> and press <b>ENTER</b>.</p>		
----------------------------------------------------------------------------------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------

▶ Continued on next page

The information shown will not fit on a single screen. You can scroll up and down to see it all.  
The statistics calculated for the data are:

mean	$\bar{x}$
sum	$\Sigma x$
sum of squares	$\Sigma x^2$
sample standard deviation	$s_x$
population standard deviation	$\sigma_x$
number	$n$
minimum value	MinX
lower quartile	$Q_1$
median	Med
upper quartile	$Q_3$
maximum value	MaxX

```
1-Var Stats
x̄=3.2
Σx=16
Σx²=96
Sx=3.346640106
σx=2.993325909
↓n=5
```

```
1-Var Stats
↑n=5
minX=1
Q1=1
Med=2
Q3=6
maxX=9
```

## 5.8 Calculating statistics from a frequency table

### Example 47

Calculate the summary statistics for this data:

<b>Number</b>	1	2	3	4	5
<b>Frequency</b>	3	4	6	5	2

Enter the data in L1 and L2 (see Example 41).

Press **STAT** | **CALC** | **1:1-Var Stats**.

Type **2nd** **L1** , **2nd** **L2** and press **ENTER**.

```
1-Var Stats L1,L2
```

The information shown will not fit on a single screen. You can scroll up and down to see it all.  
The statistics calculated for the data are:

mean	$\bar{x}$
sum	$\Sigma x$
sum of squares	$\Sigma x^2$
sample standard deviation	$s_x$
population standard deviation	$\sigma_x$
number	$n$
minimum value	minX
lower quartile	$Q_1$
median	Med
upper quartile	$Q_3$
maximum value	MaxX

```
1-Var Stats
x̄=2.95
Σx=59
Σx²=203
Sx=1.234376041
σx=1.203120942
↓n=20
```

```
1-Var Stats
↑n=20
minX=1
Q1=2
Med=3
Q3=4
maxX=5
```

### Example 48

The interquartile range is the difference between the upper and lower quartiles ( $Q_3 - Q_1$ ).

<b>Number</b>	1	2	3	4	5
<b>Frequency</b>	3	4	6	5	2

$Q_3 - Q_1$  2

Interquartile range =  $Q_3 - Q_1 = 2$

The calculator stores the values you calculate in One-Variable Statistics so that you can access them in other calculations. These values are stored until you do another One-Variable Statistics calculation.

Calculate the  $\bar{x} + \sigma_x$  for this data:

<b>Number</b>	1	2	3	4	5
<b>Frequency</b>	3	4	6	5	2

$$\bar{x} + \sigma_x = 4.15 \text{ (to 3 sf)}$$
$$\bar{x} + \sigma_x$$

4.153120942

### 5.11 The use of $nCr$

Find the value of  $\binom{8}{3}$  (or  ${}_8C_3$ ).

Press **3** **ENTER**.


```
MATH NUM CPX PRG
1:rand
2:nPr
3:nCr
4:!
5:randInt(
6:randNorm(
7:randBin(
```

▶ Continued on next page

8 nCr 3 = 56

### Example 51

List the values of  $\binom{4}{r}$  for  $r = 0, 1, 2, 3, 4$ .

Press  to display the Y= editor. The default graph type is Function, so the form Y= is displayed.

Press **4**.

Press m 3:nCr.

Press  $x, t, \theta, n$  ENTER.

```
Plot1 Plot2 Plot3
Y1=4 nCr X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

Press **2nd** **TABLE**.

The table shows that

$$\binom{4}{0}=1, \binom{4}{1}=4, \binom{4}{2}=6, \binom{4}{3}=4$$

and  $\binom{4}{4} = 1$

You may need to reset the start value and incremental values for the table using

**2nd TBLSET**

2nd TBLSET

X	Y <sub>1</sub>	
0	1	
1	4	
2	6	
3	4	
4	1	
5	0	
6	0	

Press + for  $\Delta$ [tbl]

## 5.12 Calculating binomial probabilities

### Example 52

$X$  is a discrete random variable and  $X \sim \text{Bin}(9, 0.75)$ .

Calculate  $P(X = 5)$

$$P(X=5) = \binom{9}{5} 0.75^5 0.25^4$$

The calculator can find this value directly.

Press **2nd** **DISTR** A:binompdf(

Enter 9 as trials, 0.75 as  $p$  and 5 as  $x$ .

Select Paste and press **ENTER** and x, in order.

Press **ENTER** again

You should enter  
the values:  $n$  (numtrials),  $p$   
and  $x$ , in order.

```

D:DF.DRAW
0:FCDF(
1:BINOMPDF(
B:BINOMCDF(
C:POISSONPDF(
D:POISSONCDF(
E:GEOMETPDF(
F:GEOMETCDF(

```

The calculator shows that  $P(X = 5) = 0.117$  (to 3 sf).

```
binomPdf(9,0.75,
        .1167984009
```

### Example 53

$X$  is a discrete random variable and  $X \sim \text{Bin}(7, 0.3)$ .

Calculate the probabilities that  $X$  takes the values  $\{0, 1, 2, 3, 4, 5, 6, 7\}$ .

Press **2nd** **DISTR** A:binompdf(.

Enter 7 as trials, 0.3 as  $p$  and leave  $x$  blank.

Select Paste and press **ENTER**

Press **ENTER** again

You should enter  
the values:  $n$  (numtrials),  $p$   
and  $x$ , in order.

```

DRAW
0:Fcdf(
1:binompdf(
2:binomcdf(
3:Poissonpdf(
4:Poissoncdf(
5:geometpdf(
6:geometcdf(

```

The calculator displays each of the probabilities.

To see the remaining values scroll the screen to the right.

The list can also be transferred as a list.

```

binompdf(7,0.3)
(.0823543 .247018

```

Press **STO>** **2nd** **L1** .

Press **ENTER** .

```

binompdf(7,0.3)
(.0823543 .247018
Ans→L1
(.0823543 .247018

```

Press **STAT** 1:Edit...

The binomial probabilities are now displayed in the first column.

L1	L2	L3	1
.0823543	-----	-----	
.247018			
.31765			
.22689			
.09724			
.025			
.00357			
L1(1) = .0823543			

### Example 54

$X$  is a discrete random variable and  $X \sim \text{Bin}(20, 0.45)$ .

Calculate

- the probability that  $X$  is less than or equal to 10.
- the probability that  $X$  lies between 5 and 15 inclusive.
- the probability that  $X$  is greater than 11.

Press **2nd** **DISTR** B:binomcdf(.

You are given the lower  
bound probability so you  
have to calculate other  
probabilities using this.

You should enter  
the values:  $n$  (numtrials),  
 $p$  and  $x$ , in order.

```

DRAW
0:Fcdf(
1:binompdf(
2:binomcdf(
3:Poissonpdf(
4:Poissoncdf(
5:geometpdf(
6:geometcdf(

```

► Continued on next page

<p><b>a</b> Enter 30 as trials, 0.45 as <math>p</math> and 10 as <math>x</math>.          Select Paste and press <b>ENTER</b>          Press <b>ENTER</b> again  <math>P(X \leq 10) = 0.751</math> (to 3 sf).</p>	<pre>binomcdf(20,0.4) .75071064</pre>
<p><b>b</b> <math>P(5 \leq X \leq 15) = P(X \leq 15) - P(X \leq 4)</math>          Press <b>2nd</b> <b>DISTR</b> B:binomcdf(          Enter 20 as trials, 0.45 as <math>p</math> and 10 as <math>x</math>.          Select Paste and press <b>ENTER</b>          Type <b>(-)</b> and then Press <b>2nd</b> <b>DISTR</b> B:binomcdf(          Enter 20 as trials, 0.45 as <math>p</math> and 4 as <math>x</math>.          Select Paste and press <b>ENTER</b>          Press <b>ENTER</b> again  <math>P(5 \leq X \leq 15) = 0.980</math> (to 3 sf).</p>	<pre>binomcdf(20,0.4) .9796059841</pre>
<p><b>c</b> <math>P(X &gt; 11) = 1 - P(X \leq 11)</math>          Enter <b>1</b> <b>-</b> and then Press <b>2nd</b> <b>DISTR</b> B:binomcdf(          Select Paste and press <b>ENTER</b>          Press <b>ENTER</b> again  <math>P(X &gt; 11) = 0.131</math> (to 3 sf).</p>	<pre>1-binomcdf(20,0) .130764971</pre>

## Calculating Poisson probabilities

### 5.13 Calculating Poisson probabilities

#### Example 55

<p><math>X</math> is a discrete random variable and <math>X \sim Po(0.5)</math>          Calculate  <b>i</b> <math>P(X = 2)</math>  <b>ii</b> <math>P(X \leq 2)</math>  <b>iii</b> <math>P(X &gt; 2)</math></p>	
<p><b>i</b> <math>P(X = 2) = \frac{e^{-0.5} \times (0.5)^2}{2!}</math>          The calculator can find this value directly.          Press <b>2nd</b>   <b>DISTR</b>   C:poissonpdf(          Enter the parameter and the <math>X</math> value.          Select Paste and press <b>ENTER</b></p>	<pre>Poissonpdf λ:0.5 x value:2 Paste</pre>
<p>Press <b>ENTER</b> again.          The calculator shows that  <math>P(X = 2) = 0.0758</math> (to 3 sf)</p>	<pre>Poissonpdf(0.5,) .0758163325</pre>

▶ Continued on next page

$$\text{ii } P(X \leq 2) = \frac{e^{-0.5} \times (0.5)^0}{0!} + \frac{e^{-0.5} \times (0.5)^1}{1!} + \frac{e^{-0.5} \times (0.5)^2}{2!}$$

The calculator can find this value directly.

Press **2nd** | DISTR | D:poissoncdf(

Enter the parameter and the upper bound 2.

Select Paste and press **ENTER**

```
poissoncdf
λ:0.5
x value:2
Paste
```

Press **ENTER** again.

The calculator shows that

$P(X \leq 2) = 0.986$  (to 3 sf)

```
Poissoncdf(0.5,2)
.985612322
```

$$\text{ii } P(X > 2) = \frac{e^{-0.5} \times (0.5)^3}{3!} + \frac{e^{-0.5} \times (0.5)^4}{4!} + \frac{e^{-0.5} \times (0.5)^5}{5!} + \dots$$

$$= 1 - P(X \leq 2)$$

The calculator can find this value directly.

Type 1-

Press **2nd** | DISTR | D:poissoncdf(

Enter the parameter and the upper bound 2.

Select Paste and press **ENTER**

```
poissoncdf
λ:0.5
x value:2
Paste
```

Press **ENTER** again.

The calculator shows that

$P(X > 2) = 0.0144$  (to 3 sf)

```
1-Poissoncdf(0.5,2)
.014387678
```

## Example 56

If  $X \sim \text{Po}(\lambda)$  find the value of  $\lambda$ , correct to 3 decimal places, given that  $P(X = 2) = 0.035$ .

There is no inverse Poisson function on the TI-84, so instead you could use the numerical solver function to find a value of  $\lambda$  when you are given a probability.

Press **MATH** | B: Solver...

If no equation is stored, the screen will display eqn: 0=, if not then press

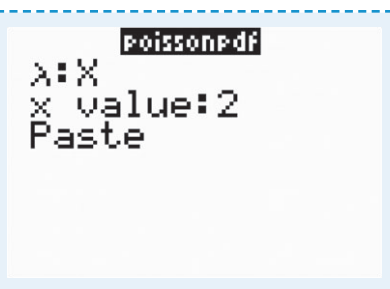
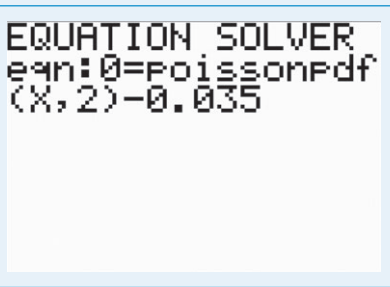
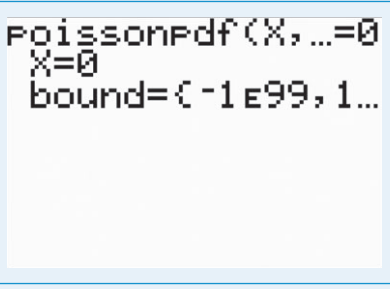
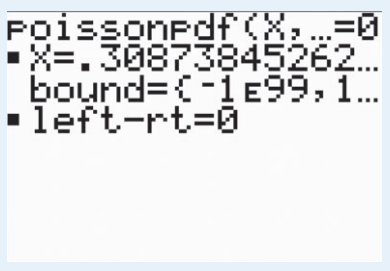
**▲** and **CLEAR**.

It is necessary to enter the variable  $x$  in the numerical solver and also an initial guess – 0 is close enough.

```
EQUATION SOLVER
eqn: 0=
```

▶ Continued on next page



Press <b>2nd</b>   DISTR   C:poissonpdf( Enter the value of $\lambda$ as $X$ , the value of $n$ as 2. Select Paste and press <b>ENTER</b>	
Type $-0.035$ and press <b>ENTER</b> This makes the equation $0 = P(X = 2) - 0.035$	
Enter an initial guess for $x - 0$ is close enough. Then press <b>ALPHA</b> SOLVE	
The required value of $\lambda$ is 0.309 (to 3 sf).	

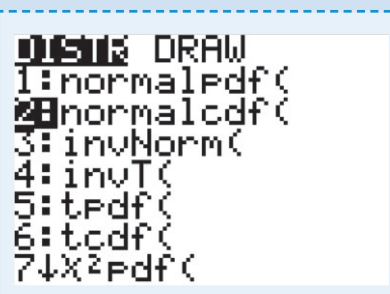
## Calculating normal probabilities

### 5.14 Calculating normal probabilities from X-values

#### Example 57

A random variable  $X$  is normally distributed with a mean of 195 and a standard deviation of 20 or  $X \sim N(195, 20^2)$ . Calculate

- the probability that  $X$  is less than 190.
- the probability that  $X$  is greater than 194.
- the probability that  $X$  lies between 187 and 196.

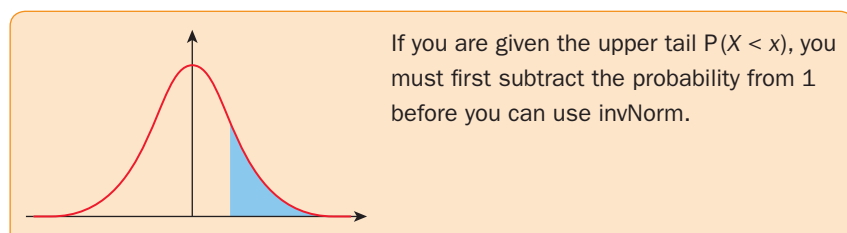
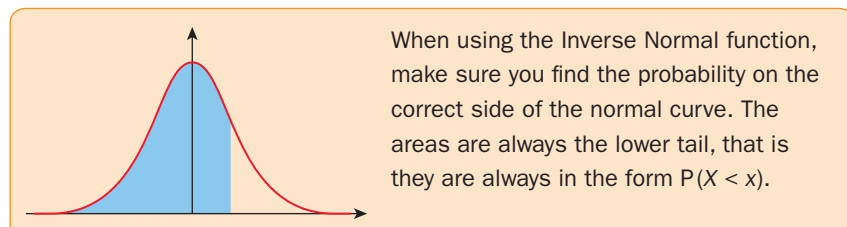
Press <b>2nd</b> <b>DISTR</b>   2:normalcdf(  Press <b>ENTER</b> .	<div> <p>You should enter the values, Lower Bound, Upper Bound, <math>\mu</math> and <math>\sigma</math>, in order.</p> </div> <div> <p>The value E99 is the largest value that can be entered in the GDC and is used in the place of <math>\infty</math>. It stands for <math>1 \times 10^{99}</math> (<math>-E99</math> is the smallest value and is used in the place of <math>-\infty</math>). To enter the E, you need to press <b>2nd</b> <b>EE</b>.</p> </div>	
--------------------------------------------------------------------------	-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	---------------------------------------------------------------------------------------

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<p><b>a</b> <math>P(X &lt; 190)</math> Enter Lower Bound as <math>-E99</math>, Upper Bound as 190, <math>\mu</math> to 195 and <math>\sigma</math> to 20. <math>P(X &lt; 190) = 0.401</math> (to 3 sf)</p>	<pre>normalcdf(-E99, 190,195,20) normalcdf(-E99, .4012937256</pre>
<p><b>b</b> <math>P(X &lt; 194)</math> Enter Lower Bound as 194, Upper Bound as <math>E99</math>, <math>\mu</math> as 195 and <math>\sigma</math> as 20. <math>P(X &gt; 194) = 0.520</math> (to 3 sf)</p>	<pre>normalcdf(194,E99, 195,20) normalcdf(194,E99, .519938874</pre>
<p><b>c</b> <math>P(187 &lt; X &lt; 196)</math> Enter Lower Bound as 187, Upper Bound as 196, <math>\mu</math> as 195 and <math>\sigma</math> as 20. <math>P(187 &lt; X &lt; 196) = 0.175</math> (to 3 sf)</p>	<pre>normalcdf(187,196, 195,20) normalcdf(187,196, .1753605711</pre>

## 5.15 Calculating X-values from normal probabilities

In some problems you are given probabilities and have to calculate the associated values of  $X$ . To do this, use the `invNorm` function.



### Example 58

A random variable  $X$  is normally distributed with a mean of 75 and a standard deviation of 12 or  $X \sim N(75, 12^2)$ .

If  $P(X < x) = 0.4$ , find the value of  $x$ .

You are given a lower-tail probability so you can find  $P(X < x)$  directly.

Press **2nd** **DISTR** | 3:invNorm(.

Press **ENTER**.

You should enter the values: area (probability),  $\mu$  and  $\sigma$ , in order.

```
0:QUIT DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7:χ²pdf(
```

Enter area (probability) as 0.4,  $\mu$  as 75 and  $\sigma$  as 12.

So if  $P(X < x) = 0.4$  then  $x = 72.0$  (to 3 sf).

```
invNorm(0.4,75,12
71.95983479
```

### Example 59

A random variable  $X$  is normally distributed with a mean of 75 and a standard deviation of 12 or  $X \sim N(75, 12^2)$ .

If  $P(X > x) = 0.2$ , find the value of  $x$ .

You are given an upper-tail probability so you must first find  $P(X < x) = 1 - 0.2 = 0.8$ . You can now use the invNorm function as before.

Press **2nd** **DISTR** | 3:invNorm(.

Press **ENTER**.

You should enter the values: area (probability),  $\mu$  and  $\sigma$ , in order.

```
0:QUIT DRAW
1:normalpdf(
2:normalcdf(
3:invNorm(
4:invT(
5:tpdf(
6:tcdf(
7:χ²pdf(
```

Enter area (probability) as 0.8,  $\mu$  as 75 and  $\sigma$  as 12.

So if  $P(X > x) = 0.2$  then  $x = 85.1$  (to 3 sf).

```
invNorm(0.8,75,12
85.0994548
```

This sketch of a normal distribution curve shows this value and the probabilities from Example 59.

